Assignment

Fundamental Definite Integral

Basic Level

1. $\int_0^{-1} e^{2\ln x} =$ **[MP PET 1990]**

(a) 0 (b) 1/2 (c) 1/3 (d) 1/4

2. $\int_1^2 e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx =$ **[MNR 1990; AMU 1999; UPSEAT 2000]**

(a) $\frac{e^2}{2}+e$ (b) $e-\frac{e^2}{2}$ (c) $\frac{e^2}{2}-e$ (d) None of these

3. $\int_2^3 \frac{dx}{x^2-x} =$ **[EAMCET 2002]**

(a) $\log\frac{2}{3}$ (b) $\log\frac{1}{4}$ (c) $\log\frac{4}{3}$ (d) $\log\frac{8}{3}$

4. $\int_1^3 (x-1)(x-2)(x-3)dx =$ **[Karnataka CET 2002]**

(a) 3 (b) 2 (c) 1 (d) 0

5. $\int_1^{\sqrt{3}} \frac{1}{1+x^2}dx$ is equal to **[DCE 2002]**

(a) $\pi/12$ (b) $\pi/6$ (c) $\pi/4$ (d) $\pi/3$

6. $\int_1^e \frac{1}{x}dx$ is equal to **[SCRA 1996]**

(a) ∞ (b) 0 (c) 1 (d) log (1 + e)

7. The value of $\int_0^{2/3} \frac{dx}{4+9x^2}$ is **[Rajasthan PET 1992; MP PET 1997]**

(a) $\pi/12$ (b) $\pi/24$ (c) $\pi/4$ (d) 0

8. $\int_0^{2\pi} \sqrt{1+\sin\frac{x}{2}}\,dx =$ **[MNR 1987; UPSEAT 2000]**

(a) 0 (b) 2 (c) 8 (d) 4

9. The value of $I=\int_0^{\pi/2} \frac{(\sin x+\cos x)^2}{\sqrt{1+\sin 2x}}dx$ is

(a) 3 (b) – 1 (c) 2 (d) 0

10. $\int_0^{\pi/2} e^x \sin x\,dx$ is equal to **[Roorkee 1978; EAMCET 1991]**

(a) $\frac{1}{2}(e^{\pi/2}-1)$ (b) $\frac{1}{2}(e^{\pi/2}+1)$ (c) $\frac{1}{2}(1-e^{\pi/2})$ (d) $2(e^{\pi/2}+1)$

11. The value of $\int_1^2 \log x\,dx$ is **[Roorkee 1995]**

(a) $\log\frac{2}{e}$ (b) log 4 (c) $\log\frac{4}{e}$ (d) log 2

12. $\int_0^1 \frac{d}{dx}\left[\sin^{-1}\left(\frac{2x}{1+x^2}\right)\right]dx$ is equal to **[Kerala (Engg.) 2002]**

(a) 0 (b) π (c) $\pi/2$ (d) $\pi/4$

13. The value of $\int_{-2}^{2}(ax^3+bx+c)dx$ depends on **[MNR 1988; Rajasthan PET 1990]**

(a) The value of *a* (b) The value of *b* (c) The value of *c* (d) The values of *a* and *b*

14. $\int_0^{\pi}\frac{dx}{1+\sin x}=$ **[CEE 1993]**

(a) 0 (b) $\frac{1}{2}$ (c) 2 (d) $\frac{3}{2}$

15. $\int_0^1 \cos^{-1}x\,dx=$ **[DSSE 1988]**

(a) 0 (b) 1 (c) 2 (d) None of these

16. If $I=\int_0^{\pi/4}\sin^2 x\,dx$ and $J=\int_0^{\pi/4}\cos^2 x\,dx$, then I = **[SCRA 1989]**

(a) $\frac{\pi}{4}-J$ (b) $2J$ (c) J (d) $\frac{J}{2}$

17. If $x(x^4+1)\phi(x)=1$, then $\int_1^2\phi(x)dx=$ **[SCRA 1986]**

(a) $\frac{1}{4}\log\frac{32}{17}$ (b) $\frac{1}{2}\log\frac{32}{17}$ (c) $\frac{1}{4}\log\frac{16}{17}$ (d) None of these

18. $\int_{1/4}^{1/2}\frac{dx}{\sqrt{x-x^2}}=$ **[SCRA 1986]**

(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

19. $\int_0^1\frac{dx}{\sqrt{1+x}-\sqrt{x}}=$ **[SCRA 1986]**

(a) $\frac{2\sqrt{2}}{3}$ (b) $\frac{4\sqrt{2}}{3}$ (c) $\frac{8\sqrt{2}}{3}$ (d) None of these

20. $\int_0^{2\pi}(\sin x+\cos x)dx=$ **[SCRA 1991]**

(a) 0 (b) 2 (c) – 2 (d) 1

21. $\int_0^3\frac{3x+1}{x^2+9}dx=$ **[EAMCET 2003]**

(a) $\log(2\sqrt{2})+\frac{\pi}{12}$ (b) $\log(2\sqrt{2})+\frac{\pi}{2}$ (c) $\log(2\sqrt{2})+\frac{\pi}{6}$ (d) $\log(2\sqrt{2})+\frac{\pi}{3}$

22. $\int_0^{\pi/4}\tan^2 x\,dx=$ **[Roorkee 1985]**

(a) $1-\frac{\pi}{4}$ (b) $1+\frac{\pi}{4}$ (c) $\frac{\pi}{4}-1$ (d) $\frac{\pi}{4}$

23. $\int_0^{\pi/2}\frac{x+\sin x}{1+\cos x}dx=$ **[MP PET 1989]**

(a) – log 2 (b) log 2 (c) $\pi/2$ (d) 0

24. If $\int_0^1 f(x)dx=M;\int_0^1 g(x)dx=N$. Which of the following is correct

(a) $\int_0^1 (f(x)+g(x))dx = M+N$ (b) $\int_0^1 (f(x)g(x))dx = MN$ (c) $\int_0^1 \frac{1}{f(x)}dx = \frac{1}{M}$ (d) $\int_0^1 \frac{f(x)}{g(x)}dx = \frac{M}{N}$

25. $\int_{-\pi/4}^{\pi/2} e^{-x} \sin x dx =$ **[CEE 1993]**

(a) $-\frac{1}{2}e^{-\pi/2}$ (b) $-\frac{\sqrt{2}}{2}e^{-\pi/4}$ (c) $-\sqrt{2}(e^{\pi/4}+e^{-\pi/4})$ (d) 0

26. $\int_1^e \frac{1+\log x}{x} dx =$ **[SCRA 1986]**

(a) $\frac{3}{2}$ (b) $\frac{1}{2}$ (c) $\frac{1}{e}$ (d) None of these

27. If $\int_0^1 x \log\left(1+\frac{x}{2}\right) dx = a + b \log \frac{2}{3}$, then **[SCRA 1986]**

(a) $a=\frac{3}{2}, b=\frac{3}{2}$ (b) $a=\frac{3}{4}, b=-\frac{3}{4}$ (c) $a=\frac{3}{4}, b=\frac{3}{2}$ (d) $a=b$

28. The value of $\int_0^{\pi/4} \frac{1+\tan x}{1-\tan x} dx$ is **[SCRA 1986]**

(a) $-\frac{1}{2}\log 2$ (b) $\frac{1}{4}\log 2$ (c) $\frac{1}{3}\log 2$ (d) None of these

29. $\int_{-1}^0 \frac{dx}{x^2+2x+2}$ is equal to **[MP PET 2000]**

(a) 0 (b) $\pi/4$ (c) $\pi/2$ (d) $-\pi/4$

30. $\int_0^{\pi/6} (2+3x^2)\cos 3x \, dx =$ **[DSSE 1985]**

(a) $\frac{1}{36}(\pi+16)$ (b) $\frac{1}{36}(\pi-16)$ (c) $\frac{1}{36}(\pi^2-16)$ (d) $\frac{1}{36}(\pi^2+16)$

31. The values of α which satisfy $\int_{\pi/2}^{\alpha} \sin x \, dx = \sin 2\alpha$ $(\alpha \in [0, 2\pi])$ are equal to **[IIT Screening]**

(a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{2}$ (c) $\frac{7\pi}{6}$ (d) All of the above

32. $\int_0^{\pi/4} (\cos x - \sin x)dx + \int_{\pi/4}^{5\pi/4} (\sin x - \cos x)dx + \int_{2\pi}^{\pi/4} (\cos x - \sin x)dx$ is equal to **[Rajasthan PET 2000]**

(a) $\sqrt{2}-2$ (b) $2\sqrt{2}-2$ (c) $3\sqrt{2}-2$ (d) $4\sqrt{2}-2$

33. If $\int_0^a x dx \le a+4$, then **[Rajasthan PET 2000]**

(a) $0 \le a \le 4$ (b) $-2 \le a \le 4$ (c) $-2 \le a \le 0$ (d) $a \le -2$ or $a \ge 4$

34. $\int_0^{\pi/2} \{x - [\sin x]\} dx$ is equal to **[AMU 1999]**

(a) $\pi^2/8$ (b) $\frac{\pi^2}{8}-1$ (c) $\frac{\pi^2}{8}-2$ (d) None of these

35. The value of $\int_0^1 \frac{x^4+1}{x^2+1} dx$ is **[MP PET 1998]**

(a) $\frac{1}{6}(3\pi-4)$ (b) $\frac{1}{6}(3-4\pi)$ (c) $\frac{1}{6}(3\pi+4)$ (d) $\frac{1}{6}(3+4\pi)$

36. If $I_m = \int_1^x (\log x)^m dx$ satisfies the relation $I_m = k - l I_{m-1}$, then

(a) $k=e$ (b) $l=m$ (c) $k=\frac{1}{e}$ (d) None of these

37. If $\int_{-1}^{4} f(x)dx = 4$ and $\int_{2}^{4}(3 - f(x))dx = 7$, then the value of $\int_{2}^{-1} f(x)dx$ is

(a) 2 (b) – 3 (c) – 5 (d) None of these

38. The value of $\int_{3}^{5} \frac{x^2}{x^2 - 4} dx$ is **[Roorkee 1992]**

(a) $2 - \log_e\left(\frac{15}{7}\right)$ (b) $2 + \log_e\left(\frac{15}{7}\right)$

(c) $2 + 4\log_e 3 - 4\log_e 7 + 4\log_e 5$ (d) $2 - \tan^{-1}\left(\frac{15}{7}\right)$

39. Let $\int_0^1 f(x)dx = 1, \int_0^1 xf(x)dx = a$ and $\int_0^1 x^2 f(x)dx = a^2$, then the value of $\int_0^1 (x-a)^2 f(x)dx =$ **[IIT 1990]**

(a) 0 (b) a^2 (c) $a^2 - 1$ (d) $a^2 - 2a + 2$

40. $\int_0^1 \frac{e^x(x-1)}{(x+1)^3} dx =$ **[SCRA 1986]**

(a) $\frac{e}{4}$ (b) $\frac{e}{4} - 1$ (c) $\frac{e}{4} + 1$ (d) None of these

41. $\int_0^{\pi/2} \frac{\cos x}{1 + \cos x + \sin x} dx =$ **[Roorkee 1989]**

(a) $\frac{\pi}{4} + \frac{1}{2}\log 2$ (b) $\frac{\pi}{4} + \log 2$ (c) $\frac{\pi}{4} - \frac{1}{2}\log 2$ (d) $\frac{\pi}{4} - \log 2$

42. $\int_{\pi/4}^{\pi/2} e^x[\log \sin x + \cot x]dx$ is equal to **[AI CBSE 1991]**

(a) $e^{\pi/4}\log 2$ (b) $-e^{\pi/4}\log 2$ (c) $\frac{1}{2}e^{\pi/4}\log 2$ (d) $-\frac{1}{2}e^{\pi/4}\log 2$

43. $\int_0^{\infty}(a^{-x} - b^{-x})dx =$ **[EAMCET 1995]**

(a) $\frac{1}{\log a} - \frac{1}{\log b}$ (b) $\log a - \log b$ (c) $\log a + \log b$ (d) $\frac{1}{\log a} + \frac{1}{\log b}$

44. $\int_0^{1/2} |\sin \pi x|\, dx$ is equal to **[DCE 1998]**

(a) 0 (b) π (c) $-\pi$ (d) $\frac{1}{\pi}$

45. $\int_0^1 \frac{x}{(1-x)^{3/4}} dx =$ **[EAMCET 1992]**

(a) $\frac{12}{5}$ (b) $\frac{16}{5}$ (c) $\frac{-16}{5}$ (d) None of these

46. $\int_1^e \log x\, dx =$ **[Karnataka CET 1994]**

(a) 1 (b) e – 1 (c) e + 1 (d) 0

47. Value of $\int_0^{1/2} \frac{dx}{\sqrt{x - x^2}}$ is **[Assam JEE 1998]**

(a) $\frac{1}{2}$ (b) π (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

48. $\int_2^4 (3x-2)^2 dx$ equals **[MP PET 1989]**

(a) 102 (b) 104 (c) 100 (d) 98

49. $\int_1^4 \left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)dx$ equals **[MP PET 1989]**

(a) 20/3 (b) 19/3 (c) 13/2 (d) 6

50. $\int_0^\infty \sec hx dx$ equals **[Karnataka CET 1996]**

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{\pi}{2}+1$ (d) 1

51. If $\frac{d}{dx}[f(x)]=\phi(x)$, then the value of $\int_1^2 \phi(x)dx$ equals **[Rajasthan PET 1995]**

(a) $f(1)-f(2)$ (b) $\phi(2)-\phi(1)$ (c) $f(2)-f(1)$ (d) $\phi(1)-\phi(2)$

52. $\int_0^a \sqrt{a^2-x^2}dx$ equals **[EAMCET 1996]**

(a) $\frac{\pi a}{4}$ (b) $\frac{\pi a^2}{4}$ (c) $\frac{\pi a^2}{2}$ (d) $\frac{\pi a}{2}$

53. $\int_0^{\pi/2} \sqrt{1+\sin 2x}\, dx$ equals **[Rajasthan PET 1990]**

(a) 1 (b) 1/2 (c) 2 (d) None of these

54. $\int_0^{\pi/4} \frac{dx}{1+\cos 2x}$ equals **[Rajasthan PET 1987]**

(a) 1 (b) – 1 (c) 1/2 (d) – 1/2

55. Let $I_1=\int_1^2 \frac{1}{\sqrt{1+x^2}}dx$ and $I_2=\int_1^2 \frac{1}{x}dx$. Then

(a) $I_1>I_2$ (b) $I_2>I_1$ (c) $I_1=I_2$ (d) $I_1>2I_2$

56. If for every integer n, $\int_n^{n+1} f(x)dx=n^2$, then the value of $\int_{-2}^4 f(x)dx$ is

(a) 16 (b) 14 (c) 19 (d) None of these

57. If $I_n=\int_0^1 x^n e^{-x}dx$ for $n\in N$, then $I_n-nI_{n-1}=$

(a) e (b) $1/e$ (c) $-1/e$ (d) None of these

58. $\int_0^3 x\sqrt{(1+x)}dx$ equals

(a) 9/2 (b) 27/4 (c) 116/15 (d) None of these

59. $\int_1^{\frac{4\sqrt{3}}{5}-1} \frac{x+2}{\sqrt{x^2+2x-3}}dx=$

(a) $\frac{2\sqrt{3}}{3}-\frac{1}{2}\log 3$ (b) $\frac{2\sqrt{3}}{3}+\frac{1}{2}\log 3$ (c) $\frac{2\sqrt{3}}{3}-\frac{1}{2}\log(\sqrt{3}+2)$ (d) $\frac{2\sqrt{3}}{3}+\frac{1}{2}\log(\sqrt{3}+2)$

60. If $I_1=\int_0^1 2^{x^2}dx, I_2=\int_0^1 2^{x^3}dx, I_3=\int_1^2 2^{x^2}dx$ and $I_4=\int_1^2 2^{x^3}dx$ then

(a) $I_1>I_2$ (b) $I_2>I_1$ (c) $I_3>I_4$ (d) $I_4>I_3$

61. If $f(x)=\begin{vmatrix} \sin x+\sin 2x+\sin 3x & \sin 2x & \sin 3x \\ 3+4\sin x & 3 & 4\sin x \\ 1+\sin x & \sin x & 1 \end{vmatrix}$, then the value of $\int_0^{\pi/2} f(x)dx$ is

(a) 3 (b) 2/3 (c) 1/3 (d) 0

62. If $\int_2^3 \frac{x^2+1}{(2x+1)(x^2-1)}dx = p\log\frac{7}{5}+q\log\frac{4}{3}+r\log 2$, then

(a) $p=-\frac{5}{6},q=1,r=\frac{1}{3}$ (b) $p=\frac{5}{6},q=1,r=\frac{1}{3}$ (c) $p=-\frac{5}{6},q=-1,r=-\frac{1}{3}$ (d) $p=\frac{5}{6},q=1,r=-\frac{1}{3}$

63. If $\frac{1}{\sqrt{a}}\int_1^a\left(\frac{3}{2}\sqrt{x}+1-\frac{1}{\sqrt{x}}\right)dx<4$, then 'a' may take values

(a) 0 (b) 4 (c) 9 (d) $\frac{13+\sqrt{313}}{2}$

64. The value of $\int_0^{\frac{\pi}{2}}\frac{\cos 3x+1}{\cos 2x-1}dx$ is

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) 0

65. $\int_0^2(t-\log_2 a)dt$ equals

(a) $\log_2(2/a)$ (b) $2\log_2(2/a)$ (c) $2\log_4(2/a)$ (d) None of these

66. $\int_0^{4/\pi}\left(3x^2\sin\frac{1}{x}-x\cos\frac{1}{x}\right)dx=$

(a) $\frac{8\sqrt{2}}{\pi^3}$ (b) $\frac{32\sqrt{2}}{\pi^3}$ (c) $\frac{24\sqrt{2}}{\pi^3}$ (d) $\frac{\sqrt{2048}}{\pi^3}$

67. The value of $\int_a^b\frac{x}{|x|}dx, a<b<0$ is **[Orissa JEE 2003]**

(a) $-(|a|+|b|)$ (b) $|b|-|a|$ (c) $|a|-|b|$ (d) $|a|+|b|$

68. The value of $\alpha\in(-\pi,0)$ satisfying $\sin\alpha+\int_\alpha^{2\alpha}\cos 2x\,dx=0$ is

(a) $-\pi/2$ (b) $-\pi$ (c) $-\pi/3$ (d) 0

69. If $\int_0^{36}\frac{1}{2x+9}dx=\log k$, then k is equal to

(a) 3 (b) 9/2 (c) 9 (d) 81

70. The value of $\lim_{x\to\pi/2}\frac{\int_{\pi/2}^x t\,dt}{\sin(2x-\pi)}$ is

(a) ∞ (b) $\pi/2$ (c) $\pi/4$ (d) $\pi/8$

71. Suppose that $f''(x)$ is continuous for all x and $f(0)=f'(1)=1$. If $\int_0^1 t f''(t)dt=0$, then the value of $f(1)$ is

(a) 2 (b) 3 (c) $4\frac{1}{2}$ (d) None of these

72. $I_{m,n}=\int_0^1 x^m(\ln x)^n dx=$ **[EAMCET 1994]**

(a) $\frac{n}{n+1}I_{m,n-1}$ (b) $\frac{-m}{n+1}I_{m,n-1}$ (c) $\frac{-n}{n+1}I_{m,n-1}$ (d) $\frac{m}{n+1}I_{m,n-1}$

Advance Level

73. If $(n-m)$ is odd and $|m|\neq|n|$, then $\int_0^\pi \cos mx\sin nx\,dx$ is

(a) $\frac{2n}{n^2-m^2}$ (b) 0 (c) $\frac{2n}{m^2-n^2}$ (d) $\frac{2m}{n^2-m^2}$

74. The value of the definite integral $\int_0^1 \frac{dx}{x^2+2x\cos\alpha+1}$ for $0<\alpha<\pi$ is equal to **[Kurukshetra CEE 2002]**

(a) $\sin\alpha$ (b) $\tan^{-1}(\sin\alpha)$ (c) $\alpha\sin\alpha$ (d) $\frac{\alpha}{2}(\sin\alpha)^{-1}$

75. If $f(y)=e^y, g(y)=y$; $y>0$ and $F(t)=\int_0^t f(t-y)g(y)dy$, then **[AIEEE 2003]**

(a) $F(t)=1-e^{-t}(1+t)$ (b) $F(t)=e^t-(1+t)$ (c) $F(t)=te^t$ (d) $F(t)=te^{-t}$

76. If $I(m,n)=\int_0^1 t^m(1+t)^n dt$, then the expression for $I(m,n)$ in terms of $I(m+1,n-1)$ is **[IIT Screening 2003]**

(a) $\frac{2^n}{m+1}-\frac{n}{m+1}I(m+1,n-1)$ (b) $\frac{n}{m+1}I(m+1,n-1)$

(c) $\frac{2^n}{m+1}+\frac{n}{m+1}I(m+1,n-1)$ (d) $\frac{m}{n+1}I(m+1,n-1)$

77. Let $f(x)$ be a function satisfying $f'(x)=f(x)$ with $f(0)=1$ and $g(x)$ be the function satisfying $f(x)+g(x)=x^2$ The value of intergral $\int_0^1 f(x)g(x)\,dx$ is equal to **[AIEEE 2003]**

(a) $\frac{1}{4}(e-7)$ (b) $\frac{1}{4}(e-2)$ (c) $\frac{1}{2}(e-3)$ (d) None of these

78. $\int_0^{\pi/2}\left(\frac{\theta}{\sin\theta}\right)^2 d\theta=$

(a) $\pi\log 2$ (b) $\frac{\pi}{\log 2}$ (c) π (d) None of these

79. The value of $\int_0^\pi |\sin^3\theta|\,d\theta$ is **[UPSEAT 2003]**

(a) 0 (b) $\frac{3}{8}$ (c) $\frac{4}{3}$ (d) π

80. $\int_0^\pi \frac{\sin\left(n+\frac{1}{2}\right)x}{\sin x}dx, (n\in N)$ is equal to **[Kurukshetra CEE 1998]**

(a) $n\pi$ (b) $(2\pi+1)\frac{\pi}{2}$ (c) π (d) 0

81. If I is the greatest of the definite integrals $I_1=\int_0^1 e^{-x}\cos^2 x\,dx$, $I_2=\int_0^1 e^{-x^2}\cos^2 x\,dx$, $I_3=\int_0^1 e^{-x^2}dx$, $I_4=\int_0^1 e^{-x^2/2}dx$, then

(a) $I=I_1$ (b) $I=I_2$ (c) $I=I_3$ (d) $I=I_4$

82. $\int_0^{\pi/2} x\cot x\,dx$ equals **[Rajasthan PET 1997]**

(a) $-\frac{\pi}{2}\log 2$ (b) $\frac{\pi}{2}\log 2$ (c) $\pi\log 2$ (d) $-\pi\log 2$

83. The value of $\lim_{x\to\infty}\frac{\left(\int_0^x e^x dx\right)^2}{\int_0^x e^{2x^2}dx}$ is

(a) 1 (b) 2 (c) 3 (d) 0

84. If $a < \int_0^{2\pi} \frac{1}{10+3\cos x} dx < b$, then the ordered pair (a, b) is

(a) $\left(\frac{2\pi}{7}, \frac{2\pi}{3}\right)$ (b) $\left(\frac{2\pi}{13}, \frac{2\pi}{7}\right)$ (c) $\left(\frac{\pi}{10}, \frac{2\pi}{13}\right)$ (d) None of these

85. Let $a_n = \int_0^{\pi/2} \cos^n x \cos nx \, dx$, then $a_{n+1} : a_n =$

(a) 1 : 2 (b) 2 : 1 (c) 1 : 1 (d) None of these

86. If $I_n = \int_0^{\pi/2} \frac{\sin^2 nx}{\sin^2 x} dx$, then

(a) $I_n = \frac{n\pi}{2}$ (b) $I_1, I_2, I_3, I_4, \dots\dots I_n \dots\dots$ are in A.P

(c) $\sin(I_{16}) = 0$ (d) All of these

87, If $n \in N$ and $\int_0^1 e^x (x-1)^n dx = 2e - 5$, then n =

(a) 1 (b) 2 (c) 3 (d) None of these

88. The value of $\int_0^{\pi/4} (\tan^n x + \tan^{n-2} x) d(x - [x])$, (where [.] denotes the greatest integer function) is

(a) $\frac{1}{n-1}$ (b) $\frac{1}{n+1}$ (c) $\frac{2}{n-1}$ (d) None of these

89. The points of intersection of $f_1(x) = \int_2^x (2t-5) dt$ and $f_2(x) = \int_0^x 2t \, dt$, are **[IIT Screening]**

(a) $\left(\frac{6}{5}, \frac{36}{25}\right)$ (b) $\left(\frac{2}{3}, \frac{4}{9}\right)$ (c) $\left(\frac{1}{3}, \frac{1}{9}\right)$ (d) $\left(\frac{1}{5}, \frac{1}{25}\right)$

90. The value of integral $\int_0^1 e^{x^2} dx$ lies in interval **[CEE 1993]**

(a) (0, 1) (b) (– 1, 0) (c) (1, e) (d) None of these

91. The greatest value of the function $f(x) = \int_1^x |t| dt$ on the interval $\left[-\frac{1}{2}, \frac{1}{2}\right]$ is given by **[IIT Screening]**

(a) $\frac{3}{8}$ (b) $-\frac{1}{2}$ (c) $-\frac{3}{8}$ (d) $\frac{2}{5}$

92. The absolute value of $\int_{10}^{19} \frac{\cos x}{1+x^8} dx$ is

(a) Less than 10^{-7} (b) More than 10^{-7} (c) Less than 10^{-6} (d) Both (a) and (c)

93. $\int_0^{\pi/4} \sin x (x - [x]) dx$ is equal to

(a) $\frac{1}{2}$ (b) $1 - \frac{1}{\sqrt{2}}$ (c) 1 (d) None of these

94. $\int_{-1}^{10} \text{sgn}\,(x - [x]) dx$ equals

(a) 10 (b) 11 (c) 9 (d) $\frac{11}{2}$

95. If $I_n = \int_0^{\pi/2} \frac{\sin(2n-1)x}{\sin x} dx$ and $a_n = \int_0^{\pi/2} \left(\frac{\sin n\theta}{\sin \theta}\right)^2 d\theta$, then $a_{n+1} - a_n =$

(a) I_n (b) $2I_n$ (c) I_{n+1} (d) 0

96. If $f'(x) = f(x) + \int_0^1 f(x)\,dx$ and given $f(0) = 1$ then $f(x) =$

(a) $\frac{e^x}{2-e} + \left(\frac{1+e}{1-e}\right)$ (b) $\frac{2e^x}{3-e} + \left(\frac{1-e}{1+e}\right)$ (c) $\frac{e^x}{2-e}$ (d) $\frac{2e^x}{3-e}$

97. On the interval $\left[\frac{5\pi}{3}, \frac{7\pi}{4}\right]$, the greatest value of the function $f(x) = \int_{5\pi/3}^{x} (6\cos t - 2\sin t)\,dt =$

(a) $3\sqrt{3} + 2\sqrt{2} + 1$ (b) $3\sqrt{3} - 2\sqrt{2} - 1$ (c) Does not exist (d) None of these

98. If $f''(x) = k$ in [0, a] then $\int_0^a f(x)dx - \left\{x f(x) - \frac{x^2}{2!} f'(x) + \frac{x^3}{3!} f''(x)\right\}_0^a =$

(a) $-\frac{ka^4}{12}$ (b) $\frac{ka^4}{24}$ (c) $-\frac{ka^4}{24}$ (d) None of these

99. The value of $\int_0^1 \frac{2^{2x+1} - 5^{2x-1}}{10^x}\,dx$ is

(a) $\frac{3}{5}\left[\frac{2}{\log_e\left(\frac{2}{5}\right)} + \frac{1}{2\log_e\left(\frac{5}{2}\right)}\right]$ (b) $-\frac{3}{5}\left[\frac{2}{\log_e\left(\frac{2}{5}\right)} + \frac{1}{2\log_e\left(\frac{5}{2}\right)}\right]$

(c) $\frac{3}{5}\left[\frac{2}{\log_e\left(\frac{2}{5}\right)} - \frac{1}{2\log_e\left(\frac{5}{2}\right)}\right]$ (d) None of these

100. If $\int_0^1 e^{x^2}(x-\alpha)\,dx = 0$, then **[MNR 1994]**

(a) $1 < \alpha < 2$ (b) $\alpha < 0$ (c) $0 < \alpha < 1$ (d) None of these

101. The value of $\lim_{x \to 0} \frac{\int_0^x \cos t^2\,dt}{x}$ is

(a) 0 (b) 1 (c) –1 (d) None of these

102. If a be a positive integer, the number of value of a satisfying $\int_0^{\pi/2} \left\{a^2\left(\frac{\cos 3x}{4} + \frac{3}{4}\cos x\right) + a\sin x - 20\cos x\right\} dx \le -\frac{a^2}{3}$ is

(a) Only one (b) Two (c) Three (d) Four

103. The expression $\frac{\int_0^n [x]\,dx}{\int_0^n \{x\}\,dx}$, where $[x]$ and $\{x\}$ are integral and fractional parts of x and $n \in N$ is equal to

(a) $\frac{1}{n-1}$ (b) $\frac{1}{n}$ (c) n (d) $n-1$

104. If $f(x) = x^3$ and $\int_a^b f(x)dx = \frac{b-a}{6}\left[f(a) + f(b) + kf\left(\frac{a+b}{2}\right)\right]$, then $k =$

(a) 0 (b) 2 (c) 4 (d) None of these

105. If $I_n = \int_0^{\pi/2} x^n \sin x\,dx$ and $n > 1$ then $I_n + n(n-1)I_{n-2}$ is equal to

(a) $n\left(\frac{\pi}{2}\right)^n$ (b) $(n-1)\left(\frac{\pi}{2}\right)^n$ (c) $n\left(\frac{\pi}{2}\right)^{n-1}$ (d) $(n-1)\left(\frac{\pi}{2}\right)^{n-1}$

106. The value of $\int_0^{\pi} e^{\sec x} \sec^3 x(\sin^2 x + \cos x + \sin x + \sin x \cos x)dx$ equals

(a) 0 (b) $e + \left(\frac{1}{e}\right)$ (c) $-e - \left(\frac{1}{e}\right)$ (d) e

107. If $f(x) = A\sin\left(\frac{\pi x}{2}\right) + B$, $f'\left(\frac{1}{2}\right) = \sqrt{2}$ and $\int_0^1 f(x)dx = \frac{2A}{\pi}$, then the constants A and B are respectively **[IIT 1995]**

(a) $\frac{\pi}{2}$ and $\frac{\pi}{2}$ (b) $\frac{2}{\pi}$ and $\frac{3}{\pi}$ (c) $\frac{4}{\pi}$ and 0 (d) 0 and $-\frac{4}{\pi}$

108. If for non-zero x, $af(x) + bf\left(\frac{1}{x}\right) = \frac{1}{x} - 5$, where $a \neq b$, then $\int_1^2 f(x)dx =$ **[IIT 1996]**

(a) $\frac{1}{(a^2+b^2)}\left[a\log 2 - 5a + \frac{7}{2}b\right]$ (b) $\frac{1}{(a^2-b^2)}\left[a\log 2 - 5a + \frac{7}{2}b\right]$

(c) $\frac{1}{(a^2-b^2)}\left[a\log 2 - 5a - \frac{7}{2}b\right]$ (d) $\frac{1}{(a^2+b^2)}\left[a\log 2 - 5a - \frac{7}{2}b\right]$

109. If $u_n = \int_0^{\pi/2} \frac{\sin^2 nx}{\sin x} dx$, then $u_2 - u_1, u_3 - u_2, u_4 - u_3, \ldots\ldots$ are in

(a) A.P. (b) G.P. (c) H.P. (d) None

110. If $f(x) = \begin{cases} x, & \text{for } x < 1 \\ x - 1, & \text{for } x \geq 1 \end{cases}$, then $\int_0^2 x^2 f(x)dx$ is equal to

(a) 1 (b) $\frac{4}{3}$ (c) $\frac{5}{3}$ (d) $\frac{5}{2}$

111. If $I = \int_0^{1/2} \frac{1}{\sqrt{1 - x^{2n}}} dx$, then

(a) $I \leq \frac{\pi}{6}$ (b) $I \geq \frac{\pi}{2}$ (c) $I \geq 0$ (d) All of these

112. If $f(x)$ and $g(x)$ are two integrable functions defined on $[a, b]$, then $\left|\int_a^b f(x)g(x)dx\right|$ is

(a) Less than $\sqrt{\left(\int_a^b f(x)dx\right)\left(\int_a^b g(x)dx\right)}$ (b) Less than or equal to $\sqrt{\left(\int_a^b f^2(x)dx\right) + \left(\int_a^b g^2(x)dx\right)}$

(c) Less than or equal to $\sqrt{\int_a^b (f^2(x)dx)\left(\int_a^b g^2(x)dx\right)}$ (d) None of these

113. If $f(x) = a + bx + cx^2$ then $\int_0^4 f(x)dx$ has the value

(a) $\frac{1}{6}\left\{f(0) + 2f\left(\frac{1}{2}\right) + f(1)\right\}$ (b) $\frac{1}{6}\left\{3f(0) + 2f\left(\frac{1}{2}\right) + 3f(1)\right\}$ (c) $\frac{1}{6}\left\{f(0) + 4f\left(\frac{1}{2}\right) + f(1)\right\}$ (d) $\frac{1}{6}\left\{f(0) + f\left(\frac{1}{2}\right) + f(1)\right\}$

Definite integral by Substitution Method

Basic Level

114. The value of $\int_0^{\pi/2} \frac{\sin x}{1 + \cos^2 x} dx$ is **[Rajasthan PET 1995]**

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

115. $\int_0^a x^2 \sin x^3\, dx$ equals **[Rajasthan PET 1995]**

(a) $(1-\cos a^3)$ (b) $3(1-\cos a^3)$ (c) $-\frac{1}{3}(1-\cos a^3)$ (d) $\frac{1}{3}(1-\cos a^3)$

116. $\int_0^\infty \frac{\log(1+x^2)}{1+x^2} dx =$

(a) $\pi \log \frac{1}{2}$ (b) $\pi \log 2$ (c) $2\pi \log \frac{1}{2}$ (d) $2\pi \log 2$

117. $\int_0^{\pi/4} \frac{dx}{\cos^4 x - \cos^2 x \sin^2 x + \sin^4 x} =$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) None of these

118. The value of the integral $\int_0^{\pi/4} \frac{\sqrt{\tan x}}{\sin x \cos x} dx$ equals **[Kurukshetra CEE 1996]**

(a) 1 (b) 2 (c) 0 (d) 4

119. $\int_1^2 \frac{1}{x^2} e^{-1/x} dx =$ **[DCE 2001]**

(a) $\sqrt{e}+1$ (b) $\sqrt{e}-1$ (c) $\frac{\sqrt{e}+1}{e}$ (d) $\frac{\sqrt{e}-1}{e}$

120. $\int_0^1 \frac{\tan^{-1} x}{1+x^2} dx =$ **[SCRA 1987; MNR 1990; Rajasthan PET 2001]**

(a) $\frac{\pi^2}{8}$ (b) $\frac{\pi^2}{16}$ (c) $\frac{\pi^2}{4}$ (d) $\frac{\pi^2}{32}$

121. The value of $\int_{1/e}^{\tan x} \frac{t\, dt}{1+t^2} + \int_{1/e}^{\cot x} \frac{dt}{t(1+t^2)} =$ **[IIT Screening]**

(a) –1 (b) 1 (c) 0 (d) None of these

122. The value of the integral $\int_{1/\pi}^{2/\pi} \frac{\sin(1/x)}{x^2} dx =$ **[IIT 1990]**

(a) 2 (b) –1 (c) 0 (d) 1

123. $\int_0^{\pi/3} \frac{\cos x}{3+4\sin x} dx =$

(a) $\frac{1}{4}\log\left(\frac{3+2\sqrt{3}}{2}\right)$ (b) $\frac{1}{2}\log\left(\frac{3+2\sqrt{3}}{2}\right)$ (c) $\frac{1}{3}\log\left(\frac{3+2\sqrt{3}}{2}\right)$ (d) None of these

124. $\int_0^1 \frac{e^{-x}}{1+e^x} dx =$ **[Roorkee 1976]**

(a) $\log\left(\frac{1+e}{e}\right)-\frac{1}{e}+1$ (b) $\log\left(\frac{1+e}{2e}\right)-\frac{1}{e}+1$ (c) $\log\left(\frac{1+e}{2e}\right)+\frac{1}{e}-1$ (d) None of these

125. The value of the integral $\int_0^{\log 5} \frac{e^x\sqrt{e^x-1}}{e^x+3} dx =$

(a) $3+2\pi$ (b) $4-\pi$ (c) $2+\pi$ (d) None of these

126. If $I_1 = \int_e^{e^2} \frac{dx}{\log x}$ and $I_2 = \int_1^2 \frac{e^x}{x} dx$, then **[Karnataka CET 2000]**

(a) $I_1 = I_2$ (b) $I_1 > I_2$ (c) $I_1 < I_2$ (d) None of these

127. $\int_0^{\pi/6} \frac{\sin x}{\cos^3 x} dx =$ **[SCRA 1979]**

(a) $\frac{2}{3}$ (b) $\frac{1}{6}$ (c) 2 (d) $\frac{1}{3}$

128. If $I_n = \int_0^{\pi/4} \tan^n \theta \, d\theta,$ then for any positive integer n, the value of $n(I_{n-1} + I_{n+1})$ is

[AIEEE 2002; Rajasthan PET 1999; Karnataka CET 2000]

(a) 1 (b) 2 (c) $\pi/4$ (d) π

129. The value of $\int_0^2 \frac{3^{\sqrt{x}}}{\sqrt{x}} dx$ is **[SCRA 1992]**

(a) $\frac{2}{\log 3}.(3^{\sqrt{2}} - 1)$ (b) 0 (c) $2.\frac{\sqrt{2}}{\log 3}$ (d) $\frac{3^{\sqrt{2}}}{\sqrt{2}}$

130. $\int_0^{\pi/2} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$ is equal to **[SCRA 1986; Karnataka CET 1999]**

(a) πab (b) $\pi^2 ab$ (c) $\frac{\pi}{ab}$ (d) $\frac{\pi}{2ab}$

131. If $I_1 = \int_0^x e^{zx} e^{-z^2} dz$ and $I_2 = \int_0^x e^{-z^2/4} dz$, then, **[MP PET 1990]**

(a) $I_1 = e^x \, I_2$ (b) $I_1 = e^{x^2} \, I_2$ (c) $I_1 = e^{x^2/2} \, I_2$ (d) None of these

132. $\int_1^x \frac{\log(x^2)}{x} dx =$ **[DCE 1999]**

(a) $(\log x)^2$ (b) $\frac{1}{2}(\log x)^2$ (c) $\frac{\log x^2}{2}$ (d) None of these

133. The value of $\int_0^1 \frac{dx}{e^x + e^{-x}}$ is **[SCRA 1980]**

(a) $\tan^{-1}\left(\frac{1-e}{1+e}\right)$ (b) $\tan^{-1}\left(\frac{e-1}{e+1}\right)$ (c) $\frac{\pi}{4}$ (d) $\tan^{-1} e - \frac{\pi}{4}$

134. $\int_0^{\pi/2} (\sin x - \cos x) \log(\sin x + \cos x) dx =$ **[SCRA 1986]**

(a) –1 (b) 1 (c) 0 (d) None of these

135. $\int_0^{\pi/2} \frac{\sin x \cos x}{\cos^2 x + 3\cos x + 2} dx$ is equal to **[MNR 1981; Rajasthan PET 1990; MP PET 1990]**

(a) $\log\left(\frac{8}{9}\right)$ (b) $\log\left(\frac{9}{8}\right)$ (c) $\log(8, 9)$ (d) None of these

136. $\int_0^{\pi/2} \frac{\cos x}{(1 + \sin x)(2 + \sin x)} dx =$ **[UPSEAT 1999]**

(a) $\log \frac{4}{3}$ (b) $\log \frac{1}{3}$ (c) $\log \frac{3}{4}$ (d) None of these

137. $\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx =$ **[Karnataka CET 1999]**

(a) $\frac{\pi}{2} - 2\log\sqrt{2}$ (b) $\frac{\pi}{2} + 2\log\sqrt{2}$ (c) $\frac{\pi}{4} - \log\sqrt{2}$ (d) $\frac{\pi}{4} + \log\sqrt{2}$

138. $\int_0^{\pi/4} \sec^7 \theta \, \sin^3 \theta \, d\theta =$

(a) $\frac{1}{12}$ (b) $\frac{3}{12}$ (c) $\frac{5}{12}$ (d) None of these

139. The correct evaluation of $\int_0^{\pi/2} \sin x \sin 2x$ is **[MP PET 1993, 2003]**

(a) $\frac{4}{3}$ (b) $\frac{1}{3}$ (c) $\frac{3}{4}$ (d) $\frac{2}{3}$

140. $\int_{\pi/4}^{\pi/2} \cos\theta \operatorname{cosec}^2\theta \, d\theta =$ **[Roorkee 1978]**

(a) $\sqrt{2}-1$ (b) $1-\sqrt{2}$ (c) $\sqrt{2}+1$ (d) None of these

141. $\int_1^2 \frac{\cos(\log x)}{x} dx =$ **[MP PET 1990]**

(a) $\sin(\log 3)$ (b) $\sin(\log 2)$ (c) $\cos(\log 3)$ (d) None of these

142. $\int_0^{\frac{1}{\sqrt{2}}} \frac{\sin^{-1} x}{(1-x^2)^{3/2}} dx =$ **[Roorkee 1984]**

(a) $\frac{\pi}{4}+\frac{1}{2}\log 2$ (b) $\frac{\pi}{4}-\frac{1}{2}\log 2$ (c) $\frac{\pi}{2}+\log 2$ (d) $\frac{\pi}{2}-\log 2$

143. $\int_0^{\pi/4} \frac{4\sin 2\theta \, d\theta}{\sin^4\theta+\cos^4\theta} =$ **[SCRA 1986]**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) π (d) None of these

144. If $I_n = \int_0^{\pi/4} \tan^n\theta \, d\theta$, then $I_8 + I_6$ equals **[Kurukshetra CEE 1996]**

(a) 1/4 (b) 1/5 (c) 1/6 (d) 1/7

145. $\int_0^1 \sqrt{\frac{1-x}{1+x}} dx$ equals **[Rajasthan PET 1997; Karnataka CET 1993; Bihar CEE 1998; AIEEE 2004]**

(a) $\left(\frac{\pi}{2}-1\right)$ (b) $\left(\frac{\pi}{2}+1\right)$ (c) $\pi/2$ (d) $(\pi+1)$

146. The value of the integral $\int_{-\pi/4}^{\pi/4} \sin^{-4} x \, dx$ is **[IIT Screening; MP PET 2003]**

(a) $\frac{3}{2}$ (b) $-\frac{8}{3}$ (c) $\frac{3}{8}$ (d) $\frac{8}{3}$

147. $\int_0^{\pi/2} \frac{dx}{2+\cos x} =$ **[BIT Ranchi 1992; Rajasthan PET 1993]**

(a) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (b) $\sqrt{3}\tan^{-1}(\sqrt{3})$ (c) $\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (d) $2\sqrt{3}\tan^{-1}(\sqrt{3})$

148. $\int_0^1 \tan^{-1} x dx =$ **[Karnataka CET 1993; Rajasthan PET 1997]**

(a) $\frac{\pi}{4}-\frac{1}{2}\log 2$ (b) $\pi-\frac{1}{2}\log 2$ (c) $\frac{\pi}{4}-\log 2$ (d) $\pi-\log 2$

149. $\int_0^{1/2} \frac{x\sin^{-1} x}{\sqrt{1-x^2}} dx$ **[IIT 1984]**

(a) $\frac{1}{2}+\frac{\sqrt{3}\pi}{12}$ (b) $\frac{1}{2}-\frac{\sqrt{3}\pi}{12}$ (c) $\frac{1}{2}-\frac{\sqrt{3\pi}}{12}$ (d) None of these

150. $\int_0^2 \sqrt{\frac{2+x}{2-x}}dx =$ **[MNR 1984; CEE 1993]**

(a) $\pi + 2$ (b) $\pi + \frac{3}{2}$ (c) $\pi + 1$ (d) None of these

151. $\int_0^1 \frac{\log x}{\sqrt{1-x^2}} dx =$ **[Ranchi BIT 1984]**

(a) $\frac{\pi}{2}\log 2$ (b) $\pi \log 2$ (c) $-\frac{\pi}{2}\log 2$ (d) $-\pi \log 2$

152. $\int_0^{\pi/2} \frac{\sin x \cos x}{1+\sin^4 x} dx =$ **[AISSE 1988]**

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{8}$

153. $\int_0^1 \frac{dx}{[ax + b(1-x)]^2} =$ **[SCRA 1986]**

(a) $\frac{a}{b}$ (b) $\frac{b}{a}$ (c) ab (d) $\frac{1}{ab}$

154. $\int_0^1 \log \sin\left(\frac{\pi}{2}x\right) dx$ is equal to **[Rajasthan PET 1997]**

(a) $-\log 2$ (b) $\log 2$ (c) $\frac{\pi}{2}\log 2$ (d) $\frac{-\pi}{2}\log 2$

155. If $I_n = \int_0^{\pi/4} \tan^n x\, dx$, then $\lim_{n\to\infty} n[I_n + I_{n-2}]$ equals **[AIEEE 2002]**

(a) 1/2 (b) 1 (c) ∞ (d) 0

156. The value of $\int_0^1 (x^3 + 3e^x + 4)(x^2 + e^x)dx$ is **[Rajasthan PET 1987]**

(a) $(3e-2)/6$ (b) $(3e+2)/6$ (c) $(3e-2)^2/36$ (d) None of these

157. $\int_{-\pi/2}^{\pi/2} \cos^3 x(1+\sin x)^2 dx$ equals **[EAMCET 1996]**

(a) 8/5 (b) 5/8 (c) -8/5 (d) -5/8

158. $\int_0^{\pi/2} \frac{dx}{1+\sin x}$ equals **[MNR 1983; Rajasthan PET 1990; Kurukshetra CEE 1997]**

(a) 0 (b) 1 (c) -1 (d) 2

159. $\int_0^{\pi/2} \frac{\sin 2x}{a\sin^2 x + b\cos^2 x} dx$ equals **[Rajasthan PET 1991]**

(a) $\frac{1}{a-b}\log\frac{b}{a}$ (b) $\frac{1}{a-b}\log\frac{a}{b}$ (c) $\frac{1}{a-b}\log(ab)$ (d) $\frac{1}{a+b}\log\frac{a}{b}$

160. If $u_n = \int_0^{\pi/4} \tan^n x\, dx$, then $u_2 + u_4, u_3 + u_5, u_4 + u_6.....$ are in **[MP PET 1990]**

(a) A.P. (b) G.P. (c) H.P. (d) None

161. $\int_1^{e^{37}} \frac{\pi \sin(\pi \ln x)}{x} dx$ is equal to

(a) 1 (b) 2 (c) e (d) 37

162. $\int_0^1 \frac{dx}{x+\sqrt{x}}$ equals **[Rajasthan PET 1993]**

(a) 0 (b) log 2 (c) log 3 (d) log 4

163. $\int_0^{\pi^2/4} \frac{\sin\sqrt{x}}{\sqrt{x}}dx$ **[Rajasthan PET 1994]**

(a) 2 (b) 1 (c) $\frac{\pi}{4}$ (d) $\frac{\pi^2}{8}$

164. $\int_2^4 \frac{\sqrt{(x^2-4)}}{x}dx =$ **[Rajasthan PET 1992]**

(a) $2(3\sqrt{3}-\pi)/3$ (b) π (c) $2(3\sqrt{3}-\pi)$ (d) None of these

165. The value of $\int_0^{\pi/2} \frac{\cos x}{1+\sin^2 x}dx$ is

(a) $\pi/4$ (b) $\pi/3$ (c) π (d) 2π

166. $\int_0^{\pi/2} \frac{dx}{1+2\sin x+\cos x}$ equals **[Rajasthan PET 1991]**

(a) $(1/2)\log 3$ (b) $\log 3$ (c) $(4/3)\log 3$ (d) None of these

167. $\int_0^1 \frac{dx}{(x^2+1)^{3/2}}$ is equal to **[Kurukshetra CEE 1991]**

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) 1 (d) $\sqrt{2}$

168. The value of $\int_0^1 \frac{x^3}{\sqrt{1-x^8}}dx$ is

(a) $\pi/2$ (b) $\pi/4$ (c) $\pi/6$ (d) $\pi/8$

169. The value of $\int_0^1 \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) 1 (d) 0

170. $\int_0^{\pi} \frac{dx}{a+b\cos x}$ equals **[Karnataka CET 1993]**

(a) $\frac{\pi}{\sqrt{a^2-b^2}}$ (b) $\frac{\pi}{ab}$ (c) $\frac{\pi}{\sqrt{a^2+b^2}}$ (d) $\pi(a+b)$

171. $\int_0^1 \frac{\sin^{-1}x}{\sqrt{1-x^2}}dx$ equals **[Rajasthan PET 1987]**

(a) $\pi^2/4$ (b) $\pi/4$ (c) $\pi^2/8$ (d) $\pi\ /8$

172. If $\int_0^1 \frac{e^t\,dt}{t+1}=a$, then $\int_{b-1}^{b} \frac{e^{-1}\,dt}{t-b-1}$ is equal to **[MP PET 1990]**

(a) ae^{-b} (b) $-ae^{-b}$ (c) $-be^{-a}$ (d) ae^{b}

173. The value of the integral $\int_{1/3}^{1} \frac{(x-x^3)^{1/3}}{x^4}dx$ is **[MP PET 1990]**

(a) 6 (b) 0 (c) 3 (d) 4

174. $\int_0^{\pi/2} \frac{\sin 2x}{a^2+b^2\sin^2 x}dx =$

(a) 0 (b) $\pi/4$ (c) $\frac{1}{b^2}\log\left(\frac{a^2+b^2}{a^2}\right)$ (d) $\frac{1}{b^2}\log\left(a^2+b^2\right)$

175. $\int_0^{\pi/2} \frac{\sin x \cos x}{1+\sin x} dx$ is

(a) $\frac{3}{2} - \log 2$ (b) $1 - \log 2$ (c) $3 - \log 2$ (d) $3 + \log 2$

176. The value of the integral $\int_2^4 \frac{\sqrt{x^2-4}}{x^4} dx$ is **[MP PET 1990]**

(a) $\sqrt{\frac{3}{32}}$ (b) $\frac{\sqrt{3}}{32}$ (c) $\frac{32}{\sqrt{3}}$ (d) $-\frac{\sqrt{3}}{32}$

177. If $\int_{\log 2}^{x} \frac{1}{\sqrt{e^x - 1}} dx = \frac{\pi}{6}$, then x is equal to **[MP PET 1990]**

(a) e^2 (b) 1/e (c) $\log 4$ (d) none of these

178. $\int_0^1 \frac{x^7}{\sqrt{1-x^4}} dx$ is equal to **[AMU 2000]**

(a) 1 (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{\pi}{3}$

179. $\int_0^{-1} \frac{(1-x^2)}{1+x^2+x^4} dx$ equals

(a) $(1/2) \log 2$ (b) $(1/2) \tan^{-1} 3$ (c) $(1/2) \log 3$ (d) none of these

180. If $\int_0^{\pi/2} \frac{d\theta}{13 - 4\sin^2\theta - 9\cos^2\theta} = \pi k$, then **[MP PET 1990]**

(a) $k = \frac{1}{3}$ (b) $k = \frac{1}{6}$ (c) $k = \frac{1}{12}$ (d) $k = \frac{1}{13}$

181. The value of $\int_1^4 \frac{dx}{x^2 - 2x + 10}$ is

(a) 0 (b) ∞ (c) $\pi/12$ (d) $\pi/6$

182. The value of $\int_0^{\pi/2} \cos x . e^{\sin x} dx$ is **[MP PET 1990]**

(a) 0 (b) 1 (c) -1 (d) $e - 1$

183. If $u_n = \int_0^{\pi/4} \tan^n x \, dx$, then $u_n + u_{n-2} =$ **[UPSEAT 2002]**

(a) $\frac{1}{n-1}$ (b) $\frac{1}{n+1}$ (c) $\frac{1}{2n-1}$ (d) $\frac{1}{2n+1}$

184. $\int_{a+c}^{b+c} f(x-c) dx$ is equal to

(a) $\int_c^b f(x-c) dx$ (b) $\int_a^b f(x) dx$ (c) $\int_a^b f(a+b+c+x) dx$ (d) none of these

185. $\int_0^{\pi/4} \frac{\cos x}{\sqrt{(1-2\sin^2 x}} dx$ is equal to

(a) $\pi/\sqrt{2}$ (b) $\pi/2\sqrt{2}$ (c) π (d) $\pi/2$

186. The value of the integral $\int_0^{\pi/4} \frac{\sin x + \cos x}{3 + \sin 2x} dx$ is **[MP PET 1990]**

(a) $\log 2$ (b) $\log 3$ (c) $\frac{1}{4} \log 3$ (d) $\frac{1}{8} \log 3$

187. $I=\int_0^{\pi/2}\frac{\sin 2x}{1+4\cos^2 x}dx$ is equal to

(a) $\frac{1}{4}\log 2$ (b) $\frac{1}{4}\log 4$ (c) $\frac{1}{4}\log 3$ (d) $\frac{1}{4}\log 5$

188. $\int_0^{\pi}\frac{dx}{1+2\sin^2 x}=$

(a) $\pi/\sqrt{3}$ (b) $\pi/3\sqrt{3}$ (c) $\pi/3$ (d) None of these

189. If $f(x)$ is a function satisfying $f\left(\frac{1}{x}\right)+x^2 f(x)=0$ for all non-zero x, then $\int_{\sin\theta}^{\mathrm{cosec}\,\theta} f(x)dx$ equals

(a) $\sin\theta+\mathrm{cosec}\,\theta$ (b) $\sin^2\theta$ (c) $\mathrm{cosec}^2\theta$ (d) 0

190. $\int_{\pi/6}^{\pi/4}\frac{1}{\sqrt{\cos x\sin^3 x}}dx$ is equal to

(a) 1 (b) $\frac{1}{3}$ (c) -2 (d) none of these

191. The tangent to the graph of the function $y=f(x)$ at the point with abscissa $x=a$ makes with x-axis an angle of $\pi/3$ and at the abscissa $x=b$ an angle of $\frac{\pi}{4}$. The value of the integral $\int_a^b f'(x)\,f''(x)dx$ is

(a) $\frac{1}{2}(1-\sqrt{3})$ (b) $\frac{1}{2}(1+\sqrt{3})$ (c) -1 (d) none

192. If $\frac{d[f(x)]}{dx}=g(x)$ for $a\le x\le b$, then $\int_a^b f(x)g(x)dx$ equals **[CEE 1993]**

(a) $f(b)-f(a)$ (b) $g(b)-g(a)$ (c) $\frac{[f(b)]^2-[f(a)]^2}{2}$ (d) $\frac{[g(b)]^2-[g(a)]^2}{2}$

193. The value of $\int_0^{\pi/2}\frac{1}{9\cos x+12\sin x}dx$ is

(a) $\frac{1}{15}\log_{10}6$ (b) $\frac{1}{15}\log_e 6$ (c) $\log\left(\frac{6}{15}\right)$ (d) none of these.

194. Let $I=\int_0^1\frac{e^x}{x+1}dx$, then the value of the integral $\int_0^1\frac{x\,e^{x^2}}{x^2+1}dx$ is

(a) I^2 (b) $\frac{1}{2}I$ (c) $2I$ (d) $\frac{1}{2}I^2$

Advance Level

195. Let $\frac{d}{dx}f(x)=\left(\frac{e^{\sin x}}{x}\right)$, $x>0$. If $\int_1^4\frac{3}{x}e^{\sin x^3}dx=f(k)-f(1)$, then one of the possible values of k is

(a) 15 (b) 16 (c) 63 (d) 64

196. $\int_8^{15}\frac{dx}{(x-3)\sqrt{x+1}}=$ **[Rajasthan PET 1991; UPSEAT 2003]**

(a) $\frac{1}{2}\log\frac{5}{3}$ (b) $\frac{1}{3}\log\frac{5}{3}$ (c) $\frac{1}{2}\log\frac{3}{5}$ (d) $\frac{1}{5}\log\frac{3}{5}$

197. $\int_0^{\pi/2} \frac{1+2\cos x}{(2+\cos x)^2}dx =$ **[CEE 1993]**

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{1}{2}$ (d) None of these

198. $\int_0^{\pi/4} \frac{\sin x+\cos x}{9+16\sin 2x}dx =$ **[IIT 1983]**

(a) $\frac{1}{20}\log 3$ (b) $\log 3$ (c) $\frac{1}{20}\log 5$ (d) None of these

199. The value of $\int_{-4}^{-5} e^{(x+5)^2}dx + 3\int_{1/3}^{2/3} e^{9(x-2/3)^2}dx$ is **[MP PET 1990]**

(a) e^5 (b) e^4 (c) $3e^2$ (d) 0

200. $\int_0^{\pi/4}[\sqrt{\tan x}+\sqrt{\cot x}]dx$ equals

(a) $\sqrt{2}\pi$ (b) $\pi/2$ (c) $\pi/\sqrt{2}$ (d) 2π

201. $\int_0^{2\pi} e^{x/2}.\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)dx =$ **[Roorkee 1982]**

(a) 1 (b) $2\sqrt{2}$ (c) 0 (d) None of these

202. $\int_0^{\pi/4}\frac{\sec x}{1+2\sin^2 x}$ is equal to **[MNR 1994]**

(a) $\frac{1}{3}\left[\log(\sqrt{2}+1)+\frac{\pi}{2\sqrt{2}}\right]$ (b) $\frac{1}{3}\left[\log(\sqrt{2}+1)-\frac{\pi}{2\sqrt{2}}\right]$ (c) $3\left[\log(\sqrt{2}+1)-\frac{\pi}{2\sqrt{2}}\right]$ (d) $3\left[\log(\sqrt{2}+1)+\frac{\pi}{2\sqrt{2}}\right]$

203. If $I(m,n)=\int_0^1 x^{m-1}(1-x)^{n-1}dx$, then

(a) $I(m,n)=\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}dx$ (b) $I(m,n)=\int_0^\infty \frac{x^{m}}{(1+x)^{m+n}}dx$ (c) $I(m,n)=\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}dx$ (d) Both (a) and (c)

204. If $I_n=\int_{\pi/4}^{\pi/2}(\tan x)^{-n}dx\ (n>1)$, then $I_n+I_{n+2}=$ **[MP PET 1990]**

(a) $\frac{1}{n-1}$ (b) $\frac{1}{n+1}$ (c) $-\frac{1}{n+1}$ (d) $\frac{1}{n}-1$

205. The value of $\int_0^\pi\left(\sum_{r=0}^{3} a_r\cos^{3-r}x\sin^r x\right)dx$ depends on **[MP PET 1990]**

(a) a_0 and a_2 (b) a_1 and a_2 (c) a_0 and a_3 (d) a_1 and a_3

206. The value of the integral $\int_0^3\frac{dx}{\sqrt{x+1}+\sqrt{5x+1}}$ is **[MP PET 1990]**

(a) $\frac{11}{15}$ (b) $\frac{14}{15}$ (c) $\frac{2}{5}$ (d) None of these

207. $\int_0^{\pi/4}\cos^{3/2}2\theta\cos\theta\, d\theta$ equal to

(a) $3/8\sqrt{2}$ (b) $3\pi/16\sqrt{2}$ (c) $3\pi/16$ (d) None of these

208. The value of the integral $\int_\alpha^\beta\sqrt{(x-\alpha)(\beta-x)}\,dx$ is **[MP PET 1990]**

(a) $\frac{\pi}{4}(\beta-\alpha)^2$ (b) $\frac{\pi}{2}(\beta-\alpha)^2$ (c) $\frac{\pi}{8}(\beta-\alpha)^2$ (d) None of these.

209. Let $I_1=\int_0^1 \frac{\tan^{-1}x}{x}dx$ and $I_2=\frac{1}{2}\int_0^{\pi/2}\frac{t}{\sin t}dt$, then **[MP PET 1990]**

(a) $I_1=I_2$ (b) $I_1<I_2$ (c) $I_1>I_2$ (d) None of these.

210. $\int_0^1 \log(\sqrt{1+x}+\sqrt{1-x})\ dx=$

(a) $\frac{1}{2}\left(\log\ 2-\frac{\pi}{2}+1\right)$ (b) $\frac{1}{2}\left(\log\ 2+\frac{\pi}{2}+1\right)$ (c) $\frac{1}{2}\left(\log\ 2+\frac{\pi}{2}-1\right)$ (d) None of these

211. Let $\frac{d}{dx}f(x)=\frac{e^{\sin x}}{x}$, $x>0$. If $\int_1^4 \frac{2e^{\sin x^2}}{x}dx=f(k)-f(I)$ then one of possible values of k is

(a) 4 (b) 16 (c) 2 (d) None of these

212. $\int_0^1 \sin\left(2\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right)dx=$ **[EAMCET 2003]**

(a) $\pi/6$ (b) $\pi/4$ (c) $\pi/2$ (d) π

213. $\int_0^\pi \frac{dx}{1-2a\cos x+a^2}$ equals **[Haryana CEE 1993; MNR 1997]**

(a) $\frac{\pi}{2(1-a^2)}$ (b) $\pi(1-a^2)$ (c) $\frac{\pi}{1-a^2}$ (d) None of these

Properties of Definite Integration

Basic Level

214. $\int_0^1 f(1-x)\ dx$ has the same value as the integral **[SCRA 1990]**

(a) $\int_0^1 f(x)\ dx$ (b) $\int_0^1 f(-x)\ dx$ (c) $\int_0^1 f(x-1)\ dx$ (d) $\int_{-1}^1 f(x)\ dx$

215. $\left[\sum_{n=1}^{10}\int_{-2n-1}^{2n}\sin^{27}x\ dx\right]+\left[\sum_{n=1}^{10}\int_{-2n}^{2n+1}\sin^{27}x\ dx\right]$ equals **[MP PET 2002]**

(a) 27^2 (b) –54 (c) 36 (d) 0

216. Let a, b, c bc non-zero real numbers such that $\int_0^3(3ax^2+2bx+c)\ dx=\int_1^3(3ax^2+2bx+c)\ dx$, then

(a) $a+b+c=3$ (b) $a+b+c=1$ (c) $a+b+c=0$ (d) $a+b+c=2$

217. $\int_0^1 |\ \sin\ 2\pi x\ |\ dx$ is equal to

(a) 0 (b) $-\frac{1}{\pi}$ (c) $\frac{1}{\pi}$ (d) $\frac{2}{\pi}$

218. $\int_0^2 |\ x-1|\ dx=$ **[UPSEAT 2003]**

(a) 0 (b) 2 (c) 1/2 (d) 1

219. $\int_{-1}^{2} |x|\, dx =$ **[DCE 1999]**

(a) 5/2 (b) 1/2 (c) 3/2 (d) 7/2

220. $\int_{0}^{3} |2-x|\, dx =$ **[Rajasthan PET 1999]**

(a) 2/7 (b) 5/2 (c) 3/2 (d) $-3/2$

221. The value of $\int_{1}^{5} (|x-3| + |1-x|)\, dx$ is **[IIT Screening]**

(a) 10 (b) $\frac{5}{6}$ (c) 21 (d) 12

222. $\int_{1/e}^{e} |\log x|\, dx =$ **[UPSEAT 2001]**

(a) $1-\frac{1}{e}$ (b) $2\left(1-\frac{1}{e}\right)$ (c) $e^{-1}-1$ (d) none of these

223. The correct evaluation of $\int_0^{\pi/2}\left|\sin\left(x-\frac{\pi}{4}\right)\right|dx$ is **[MP PET 1993]**

(a) $2+\sqrt{2}$ (b) $2-\sqrt{2}$ (c) $-2+\sqrt{2}$ (d) 0

224. The value of $\int_0^1 |3x^2-1|\, dx$ is **[AMU 1999]**

(a) 0 (b) $4/3\sqrt{3}$ (c) 3/7 (d) 5/6

225. $\int_0^{\pi/2} |\sin x-\cos x|\, dx =$ **[Roorkee 1990; MP PET 2001; UPSEAT 2001]**

(a) 0 (b) $2(\sqrt{2}-1)$ (c) $\sqrt{2}-1$ (d) $2(\sqrt{2}+1)$

226. $\int_0^2 |(1-x)|\, dx =$ **[SCRA 1990; Rajasthan PET 2001]**

(a) 1 (b) 2 (c) 3 (d) 0

227. $\int_{-4}^{4} |x+2|\, dx =$

(a) 50 (b) 24 (c) 20 (d) None of these

228. $\int_0^{2\pi} |\sin x|\, dx =$

(a) 0 (b) 1 (c) 2 (d) 4

229. Let $f(x)=x-[x]$, for every real number x, where $[x]$ is the integer part of x, then $\int_{-1}^{1} f(x)dx$ is

(a) 1 (b) 2 (c) 0 (d) 1/2

230. Find the value of $\int_0^9 [\sqrt{x}+2]dx$, where [.] is the greatest integer function **[UPSEAT 2002]**

(a) 31 (b) 22 (c) 23 (d) None of these

231. If $[x]$denotes the greatest integer less than or equal to x, then the value of the integral $\int_0^2 x^2[x]dx$ is

(a) 5/3 (b) 7/3 (c) 8/3 (d) 4/3

232. The value of $I=\int_0^1 x\left|x-\frac{1}{2}\right| dx$ is

(a) 1/3 (b) 1/4 (c) 1/8 (d) None of these

233. The value of $\int_0^{\sqrt{2}} [x^2]\, dx$ where [.] is the greatest integer function

(a) $2-\sqrt{2}$ (b) $2+\sqrt{2}$ (c) $\sqrt{2}-1$ (d) $\sqrt{2}-2$

234. $\int_0^{2\pi} (\sin x+|\sin x|)\, dx =$ **[Karnataka CET 2003]**

(a) 0 (b) 4 (c) 8 (d) 1

235. $\int_0^{\pi} |\sin x+\cos x|\, dx$ is equal to **[WB JEE 1994]**

(a) $\sqrt{2}$ (b) 2 (c) $2\sqrt{2}$ (d) $1/\sqrt{2}$

236. The value of integral $\int_{-2}^{4} x[x]dx$ is

(a) $\frac{41}{2}$ (b) 20 (c) $\frac{21}{2}$ (d) None of these.

237. The value of $\int_{-2}^{3} |1-x^2|\, dx$ is

(a) $\frac{1}{3}$ (b) $\frac{14}{3}$ (c) $\frac{7}{3}$ (d) $\frac{28}{3}$

238. If $a < 0 < b$, then $\int_{a}^{b} x|x|\, dx =$

(a) $\frac{1}{2}(a^2+b^2)$ (b) $\frac{1}{3}(b^2-a^2)$ (c) $\frac{1}{3}(a^3+b^3)$ (d) None of these

239. The value of $\int_{-1}^{3}(|x-2|+[x])dx$ is ($[x]$ stands for greatest integer less than or equal to x)

(a) 7 (b) 5 (c) 4 (d) 3

240. $\int_0^{\pi/2} \frac{d\theta}{1+\tan\theta}$ is equal to **[Roorkee 1980; Karnataka CET 1993; MP PET 1996; DCE 1999]**

(a) π (b) $\pi/2$ (c) $\pi/3$ (d) $\pi/4$

241. $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\cot x}}$ is **[DCE 2001]**

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{12}$ (d) $\frac{\pi}{2}$

242. $\int_0^{\pi} \frac{x\ \tan\ x}{\sec\ x+\cos\ x} dx =$ **[MNR 1985; BIT Ranchi 1986; UPSEAT 2002]**

(a) $\frac{\pi^2}{4}$ (b) $\frac{\pi^2}{2}$ (c) $\frac{3\pi^2}{2}$ (d) $\frac{\pi^2}{3}$

243. The value of $\int_0^{\pi/2} \frac{2^{\sin x}}{2^{\sin x}+2^{\cos x}} dx$ is **[Karnataka CET 1999]**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) π (d) 2π

244. $\int_0^{\pi} \frac{x\tan x}{\sec x+\tan x}$ is equal to **[MNR 1984]**

(a) $\frac{\pi}{2}-1$ (b) $\pi\left(\frac{\pi}{2}+1\right)$ (c) $\frac{\pi}{2}+1$ (d) $\pi\left(\frac{\pi}{2}-1\right)$

245. The value of $\int_0^1 \frac{dx}{x+\sqrt{1-x^2}}$ is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{1}{2}$ (d) $\frac{\pi}{4}$

246. The value of $\int_0^{\pi/2} \frac{dx}{1+\tan^3 x}$ is **[IIT 1993; DCE 2001]**

(a) 0 (b) 1 (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

247. $\int_0^{\pi} x\sin x\, dx =$ **[SCRA 1980, 1991]**

(a) π (b) 0 (c) 1 (d) π^2

248. $\int_0^{\pi/2} \frac{1}{1+\sqrt{\tan x}} dx =$ **[Rajasthan PET 1995; Kurukshetra CEE 1998]**

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) 1

249. $\int_0^{\pi/2} (a\cos^2 x + b\sin^2 x)dx$ equals **[Ranchi BIT 1994]**

(a) $(a+b)\pi/4$ (b) $(a+b)\pi/2$ (c) $(a+b)\pi/3$ (d) None of these

250. $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx$ equals **[Rajasthan PET 1996; Kerala (Engg.) 2002]**

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

251. $\int_0^{2a} \frac{f(x)}{f(x)+f(2a-x)} dx =$ **[Haryana CEE 1997; Assam JEE 1999; IIT 1999; Karnataka CET 2000]**

(a) a (b) $\frac{a}{2}$ (c) $2a$ (d) 0

252. $\int_0^{\pi/2} \frac{\cos x - \sin x}{1+\sin x \cos x} dx =$

(a) 2 (b) –2 (c) 0 (d) None of these

253. $\int_0^{\pi} |\cos x|\, dx =$ **[MP PET 1998]**

(a) π (b) 0 (c) 2 (d) 1

254. $\int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx =$ **[MNR 1989; UPSEAT 2002]**

(a) 0 (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) None of these

255. $\int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x}+\sqrt{\tan x}} dx =$ **[MP PET 1990, 1995; IIT 1983; MNR 1990]**

(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

256. $\int_0^{\pi/2} \log \sin x\, dx =$ **[MP PET 1994; Rajasthan PET 1995, 96, 97]**

(a) $-\left(\frac{\pi}{2}\right)\log 2$ (b) $\pi \log \frac{1}{2}$ (c) $-\pi \log \frac{1}{2}$ (d) $\frac{\pi}{2}\log 2$

257. The value of $\int_0^{\pi/2} \frac{e^{x^2}}{e^{x^2}+e^{\left(\frac{\pi}{2}-x\right)^2}} dx$ is **[AMU 1999]**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $e^{\pi^2/16}$ (d) $e^{\pi^2/4}$

258. The maximum and minimum value of the integral $\int_0^{\pi/2} \frac{dx}{(1+\sin^2 x)}$ are

(a) $\frac{\pi}{4}$ (b) π (c) $\frac{\pi}{2}$ (d) Both (a) and (c)

259. $\int_0^{\pi/2} \log \tan x\, dx$ **[MP PET 1999; Rajasthan PET 2001, 02; Karnataka CET 1999, 2000, 01, 02]**

(a) $\frac{\pi}{2}\log_e^2$ (b) $-\frac{\pi}{2}\log_e^2$ (c) $\pi \log_e^2$ (d) 0

260. $\int_0^{\pi/2} \sin 2x \log \tan x \, dx =$ **[Kerala (Engg.) 2002; AI CBSE 1990; Karnataka CET 1996, 98]**

(a) 1 (b) –1 (c) 0 (d) None of these

261. $\int_0^{\pi/4} \log(1+\tan x)\,dx =$ **[SCRA 1986; Karnataka CET 2000; IIT 1997]**

(a) $\frac{\pi}{4}\log 2$ (b) $\frac{\pi}{4}\log\frac{1}{2}$ (c) $\frac{\pi}{8}\log 2$ (d) $\frac{\pi}{8}\log\frac{1}{2}$

262. $\int_0^{2\pi} \frac{\sin 2\theta}{a-b\cos\theta}\,d\theta =$ **[Roorkee 1988]**

(a) 1 (b) 2 (c) $\frac{\pi}{4}$ (d) 0

263. $\int_0^{\pi} x \sin^3 x \, dx =$ **[CEE 1993]**

(a) $\frac{4\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) 0 (d) None of these

264. If f and g are continuous function on $[0, a]$ satisfying $f(x) = f(a-x)$ and $g(x) + g(a-x) = 2$ then $\int_0^a f(x)g(x)\,dx =$ **[IIT 1989]**

(a) $\int_0^a f(x)\,dx$ (b) $\int_a^0 f(x)\,dx$ (c) $2\int_0^a f(x)\,dx$ (d) None of these

265. If $\int_0^{\pi} x f(\cos^2 x + \tan^2 x)\,dx = k\int_0^{\pi/2} f(\cos^2 x + \tan^4 x)\,dx$, then the value of k is

(a) $\frac{\pi}{2}$ (b) π (c) $-\frac{\pi}{2}$ (d) None of these

266. The value of $\int_{\pi/4}^{3\pi/4} \frac{\phi}{1+\sin\phi}\,d\phi$, is **[AI CBSE 1990; IIT 1993]**

(a) $\pi \tan\frac{\pi}{8}$ (b) $\log\tan\frac{\pi}{8}$ (c) $\tan\frac{\pi}{8}$ (d) None of these

267. The value of $\int_0^{\pi} e^{\cos^2 x}\cos^5 3x\,dx$ is **[Bihar CEE 1994]**

(a) 1 (b) –1 (c) 0 (d) None of these

268. If $\int_{-1}^{1} f(x)\,dx = 0$, then **[SCRA 1990]**

(a) $f(x) = f(-x)$ (b) $f(-x) = -f(x)$ (c) $f(x) = 2f(x)$ (d) None of these

269. $\int_{-\alpha}^{\alpha} f(x)dx =$ **[MP PET 1994]**

(a) $2\int_0^{\alpha} f(x)dx$ (b) $\int_{-\alpha}^{\alpha} f(-x)dx$ (c) 0 (d) None of these

270. The value $\int_{-2}^{2}\left[p\ln\left(\frac{1+x}{1-x}\right) + q\ln\left(\frac{1-x}{1+x}\right)^{-2} + r\right]dx$ depends on **[Orissa JEE 2003]**

(a) The value of p (b) The value of q (c) the value of r (d) The value of p and q

271. $\int_{-\pi/2}^{\pi/2} \log\left(\frac{2-\sin x}{2+\sin x}\right)dx =$

(a) 0 (b) 1 (c) 2 (d) None of these

272. $\int_{-1}^{1} \log\left(\frac{1+x}{1-x}\right)dx =$ **[MP PET 1995]**

(a) 2 (b) 1 (c) 0 (d) π

273. To find the numerical value of $\int_{-2}^{2}(px^2+qx+s)\,dx$ it is necessary to know the values of constants **[IIT 1992]**

(a) p (b) q (c) s (d) p and s

274. $\int_{-a}^{a}\sin x\, f(\cos x)dx =$ **[Rajasthan PET 1997]**

(a) $2\int_{0}^{a}\sin x\, f(\cos x)dx$ (b) 0 (c) 1 (d) None of these

275. $\int_{-\pi/2}^{\pi/2}\frac{\sin x}{1+\cos^2 x}e^{-\cos^2 x}dx$ is equal to **[AMU 1999]**

(a) $2e^{-1}$ (b) 1 (c) 0 (d) None of these

276. $\int_{-3}^{3}\frac{x^2\sin 2x}{x^2+1}dx =$

(a) 0 (b) 1 (c) $2\log_e 3$ (d) None of these

277. $\int_{-\frac{1}{2}}^{\frac{1}{2}}\cos x\ln\frac{1+x}{1-x}dx$ is equal to **[AMU 2000; MNR 1998]**

(a) 0 (b) 1 (c) 2 (d) ln 3

278. $\int_{-1/2}^{1/2}(\cos x)\left[\log\left(\frac{1-x}{1+x}\right)\right]dx =$ **[Karnataka CET 2002]**

(a) 0 (b) 1 (c) $e^{1/2}$ (d) $2e^{1/2}$

279. The value of $\int_{-\pi/2}^{\pi/2}(3\sin x+\sin^3 x)dx$ is **[MP PET 2003]**

(a) 3 (b) 2 (c) 0 (d) $\frac{10}{3}$

280. $\int_{-1}^{1}\log\frac{2-x}{2+x}dx =$ **[Roorkee 1986; Kurukshetra CEE 1998]**

(a) 2 (b) 1 (c) –1 (d) 0

281. $\int_{-1}^{1}x|x|\,dx =$ **[MP PET 1990]**

(a) 1 (b) 0 (c) 2 (d) –2

282. $\int_{-2}^{2}|x|\,dx =$ **[MP PET 2000]**

(a) 0 (b) 1 (c) 2 (d) 4

283. The value of $\int_{-1}^{1}(\sqrt{1+x+x^2}-\sqrt{1-x+x^2})dx$ is

(a) 0 (b) 1 (c) –1 (d) None of these

284. $\int_{-1}^{1}\sin^3 x\cos^2 x\,dx =$ **[MNR 1991; UPSEAT 2000]**

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) 2

285. $\int_{-1}^{1}\sin^{11}x\,dx$ is equal to **[MNR 1995]**

(a) $\frac{10}{11}.\frac{8}{9}.\frac{6}{7}.\frac{4}{5}.\frac{2}{3}$ (b) $\frac{10}{11}.\frac{8}{9}.\frac{6}{7}.\frac{4}{5}.\frac{2}{3}.\frac{\pi}{2}$ (c) 1 (d) 0

286. If. $f:R\to R$ and $g:R\to R$ are one to one, real valued functions, then the value of the integral $\int_{-\pi}^{\pi}[f(x)+f(-x)][g(x)-g(-x)]dx$ is

[DCE 2001]

(a) 0 (b) π (c) 1 (d) None of these

287. $\int_{-1}^{1} x \tan^{-1} x\, dx$ equals [Rajasthan PET 1997]

(a) $\left(\frac{\pi}{2}-1\right)$ (b) $\left(\frac{\pi}{2}+1\right)$ (c) $(\pi-1)$ (d) 0

288. If $f(x)$ is an odd function of x, then $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(\cos x)\, dx$ is equal to [MP PET 1998]

(a) 0 (b) $\int_0^{\frac{\pi}{2}} f(\cos x)dx$ (c) $2\int_0^{\frac{\pi}{2}} f(\sin x)dx$ (d) $\int_0^{\pi} f(\cos x)dx$

289. $\int_{-\pi}^{\pi} (\cos px - \sin qx)^2 dx$ is equal to (where p and q are integers) [IIT 1992]

(a) $-\pi$ (b) 0 (c) π (d) 2π

290. The value of $\int_{-1}^{1} \frac{\sin x - x^2}{3-|x|} dx$ [Roorkee 1995]

(a) 0 (b) $2\int_0^1 \frac{\sin x}{3-|x|} dx$ (c) $2\int_0^1 \frac{-x^2}{3-|x|} dx$ (d) $2\int_0^1 \frac{\sin x - x^2}{3-|x|} dx$

291. $\int_{-\pi/2}^{\pi/2} \sqrt{\frac{1}{2}(1-\cos 2x)}dx =$

(a) 0 (b) 2 (c) 1/2 (d) None of these

292. $\int_{-1}^{1} x^{17} \cos^4 x\, dx =$ [MP PET 1990]

(a) – 2 (b) – 1 (c) 0 (d) 2

293. If $f(x)=\frac{e^x}{1+e^x}, I_1 = \int_{f(-a)}^{f(a)} xg\{x(1-x)\}dx$ and $I_2 = \int_{f(-a)}^{f(a)} g\{x(1-x)\}dx$, then the value of $\frac{I_2}{I_1}$ is [AIEEE 2004]

(a) 1 (b) – 3 (c) – 1 (d) 2

294. $\int_{-\pi/2}^{\pi/2} \sqrt{(\cos x - \cos^3 x)}dx =$ [Rajasthan PET 1991]

(a) $\frac{3}{4}$ (b) $-\frac{3}{4}$ (c) $\frac{4}{3}$ (d) 0

295. Let m be any integer. Then the integral $\int_0^{\pi} \frac{\sin 2m x}{\sin x} dx$ equals

(a) 0 (b) π (c) 1 (d) None of these

296. $\int_0^{2a} f(x)dx =$ [Rajasthan PET 2002]

(a) $2\int_0^a f(x)dx$ (b) 0 (c) $\int_0^a f(x)dx + \int_0^a f(2a-x)dx$ (d) $\int_0^a f(x)dx + \int_0^{2a} f(2a-x)dx$

297. If $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx$, then [SCRA 1986]

(a) $f(2a-x)=-f(x)$ (b) $f(2a-x)=f(x)$ (c) $f(a-x)=-f(x)$ (d) $f(a-x)=f(x)$

298. $\int_0^{\pi} \frac{\cos x\, dx}{[\cos(x/2)+\sin(x/2)]^3}$ equals

(a) 1 (b) – 1 (c) 0 (d) 2

299. The value of $\int_0^{2\pi} \cos^{99} x\, dx$ is

(a) 1 (b) – 1 (c) 99 (d) 0

300. The value of $\int_0^{2\pi} |\sin^3 x|\, dx$ is

(a) 0 (b) 3/8 (c) 8/3 (d) π

301. $\int_0^{\pi} \log \sin^2 x dx =$ **[MP PET 1997]**

(a) $2\pi \log_e \left(\frac{1}{2}\right)$ (b) $\pi \log_e 2 + c$ (c) $\frac{\pi}{2} \log_e \left(\frac{1}{2}\right) + c$ (d) None of these

302. $\int_0^{2\pi} \sin^5 x\, dx$ equals

(a) 0 (b) 16/15 (c) 32/15 (d) None of these

303. The value of $\int_0^1 \log\left(\frac{1}{x} - 1\right) dx$ is **[Rajasthan PET 1988]**

(a) 1 (b) 0 (c) $\pi/4$ (d) None of these

304. $\int_0^{\pi/2} \frac{\sin 8x \log(\cot x)}{\cos 2x} dx$ equals

(a) 0 (b) $\pi/4$ (c) $\pi/2$ (d) None of these

305. $\int_0^{\pi} \sin 2x \sin^3 x\, dx$ is equal to **[RPET 1993]**

(a) 0 (b) 1 (c) 2 (d) 4

306. If $f(x) = f(2 - x)$, then $\int_{0.5}^{1.5} x f(x) dx$ equals **[AMU 1999]**

(a) $\int_0^1 f(x) dx$ (b) $\int_{0.5}^{1.5} f(x) dx$ (c) $2\int_{0.5}^{1.5} f(x) dx$ (d) 0

307. The value of the integral $\int_{\frac{1}{n}}^{\frac{an-1}{n}} \frac{\sqrt{x}}{\sqrt{a-x} + \sqrt{x}} dx$ is **[AMU 2002]**

(a) $\frac{a}{2}$ (b) $\frac{na+2}{2n}$ (c) $\frac{na-2}{2n}$ (d) None of these

308. Let $I_1 = \int_a^{\pi-a} x f(\sin x) dx, I_2 = \int_a^{\pi-a} f(\sin x) dx$, then I_2 is equal to **[AMU 2000]**

(a) $\frac{\pi}{2} I_1$ (b) πI_1 (c) $\frac{2}{\pi} I_1$ (d) $2I_1$

309. Let f be a positive function. Let $I_1 = \int_{1-k}^{k} x\, f\{x(1-x)\} dx$, $I_2 = \int_{1-k}^{k} f\{x(1-x)\} dx$ where $2k - 1 > 0$. Then I_1 / I_2 is **[IIT 1997]**

(a) 2 (b) k (c) 1/2 (d) 1

310. The value of $\int_2^3 \frac{dx}{\sqrt{(x-2)(3-x)}}$ is

(a) 1 (b) $\pi/2$ (c) π (d) 2π

311. If $I = \int_0^{100\pi} \sqrt{(1 - \cos 2x)} dx$, then the value of I is

(a) $100\sqrt{2}$ (b) $200\sqrt{2}$ (c) $50\sqrt{2}$ (d) None of these

312. $\int_{\pi}^{10\pi} |\sin x|\, dx$ is **[AIEEE 2002]**

(a) 20 (b) 8 (c) 10 (d) 18

313. $\int_0^{1000} e^{x-[x]}dx$ is **[AMU 2002]**

(a) $e^{1000}-1$ (b) $\frac{e^{1000}-1}{e-1}$ (c) $1000(e-1)$ (d) $\frac{e-1}{1000}$

314. If n is a positive integer and $[x]$ is the greatest integer not exceeding x, then $\int_0^{x}\{x-[x]\}dx$ equals

(a) $n^2/2$ (b) $n(n-1)/2$ (c) $n/2$ (d) $\frac{n^2}{2}-n$

315. The value of $\int_{-\frac{\pi}{2}}^{199\frac{\pi}{2}} \sqrt{(1+\cos 2x)}dx$ equals **[AMU 1999]**

(a) $50\sqrt{2}$ (b) $100\sqrt{2}$ (c) $150\sqrt{2}$ (d) $200\sqrt{2}$

316. If $\int_0^{50\pi}(\sin^4 x+\cos^4 x)dx = k\int_0^{\pi/2}\left(\frac{3}{4}+\frac{1}{4}\cos 4x\right)dx$, then $k=$

(a) 200 (b) 100 (c) 50 (d) 25

317. $\int_0^{\pi} xf(\sin x)\,dx=$ **[IIT 1982; Kurukshetra CEE 1993]**

(a) $\pi\int_0^{\pi}f(\sin x)dx$ (b) $\frac{\pi}{2}\int_0^{\pi}f(\sin x)dx$ (c) $\frac{\pi}{2}\int_0^{\pi/2}f(\sin x)dx$ (d) None of these

318. If $\int_0^{\pi}xf(\sin x)dx = A\int_0^{\pi/2}f(\sin x)dx$, then A is **[AIEEE 2004]**

(a) 2π (b) π (c) $\frac{\pi}{4}$ (d) 0

319. The value of the definite integral $\int_0^1 \frac{x\,dx}{x^3+16}$ lies in the interval $[a, b]$. The smallest such interval is

(a) $\left[0,\frac{1}{17}\right]$ (b) $[0, 1]$ (c) $\left[0,\frac{1}{27}\right]$ (d) None of these

320. If $f(x)$ is a periodic function with period T, then

(a) $\int_a^b f(x)dx=\int_a^{b+T}f(x)dx$ (b) $\int_a^b f(x)dx=\int_{a+T}^{b}f(x)dx$ (c) $\int_a^b f(x)dx=\int_{a+T}^{b+T}f(x)dx$ (d) $\int_a^b f(x)dx=\int_{a+T}^{b+2T}f(x)dx$

321. If $f(x)$ is an odd function defined on $[-T/2, T/2]$ and has period T, then $\phi(x)=\int_a^x f(t)\,dt$ is

(a) A periodic function with period $T/2$ (b) A periodic function with period T

(c) Not a periodic function (d) A periodic function with period $T/4$

Advance Level

322. If $[x]$ denotes the greatest integer less than or equal to x, then the value of $\int_1^5 [|x-3|]\,dx$ is

(a) 1 (b) 2 (c) 4 (d) 8

323. $\int_{-2}^{2}|[x]|\,dx=$ **[EAMCET 2003]**

(a) 1 (b) 2 (c) 3 (d) 4

324. If for a real number y, $[y]$ is the greatest integer less than or equal to y, then the value of the integral $\int_{\pi/2}^{3\pi/2}[2\sin x]\,dx$ is

[IIT 1999]

(a) $-\pi$ (b) 0 (c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

325. The value of $\int_{\pi}^{2\pi} [2 \sin x] \, dx$, where [.] represents the greatest integer function is [IIT 1995]

(a) $-\pi$ (b) -2π (c) $-\frac{5\pi}{3}$ (d) $\frac{5\pi}{3}$

326. If $f(x) = \min \{|x-1|, |x|, |x+1|\}$, then $\int_{-1}^{1} f(x) \, dx$ equals

(a) 1 (b) 0 (c) 2 (d) None of these

327. The value of $\int_0^2 [x^2 - x + 1] \, dx$, (where [x] denotes the greatest integer function) is given by

(a) $\frac{5-\sqrt{5}}{2}$ (b) $\frac{6-\sqrt{5}}{2}$ (c) $\frac{7-\sqrt{5}}{2}$ (d) $\frac{8-\sqrt{5}}{2}$

328. $\int_{-2}^{2} \min(x-[x], -x-[-x]) \, dx$ equals ([x] represent greatest integer less than or equal to x)

(a) 2 (b) 1 (c) 4 (d) 0

329. Let a, b, c be non-zero real numbers such that $\int_0^{-1} (1+\cos^8 x)(ax^2+bx+c)dx = \int_0^2 (1+\cos^8 x)(ax^2+bx+c)dx$. Then the quadratic equation $ax^2+bx+c=0$ has

(a) No root in (0, 2) (b) At least one root in (0, 2) (c) A double root in (0, 2) (d) None of these

330. $\int_0^{\pi/2} f(\sin 2x) \sin x \, dx = \int_0^{\pi/2} f(\sin 2x) \cos x \, dx = \sqrt{2} \int_0^{\pi/4} f(\cos 2x) \cos x \, dx$ [IIT 1996]

(a) Question is true (b) Question is false (c) Some data is missing (d) None of these

331. If $g(x) = \int_0^x \cos^4 t \, dt$ then $g(x+\pi)$ equals [IIT 1997 Re-Exam; DCE 2001; UPSEAT 2001]

(a) $g(x)+g(\pi)$ (b) $g(x)-g(\pi)$ (c) $g(x).g(\pi)$ (d) $g(x)/g(\pi)$

332. $\int_0^{\pi} \frac{x \, dx}{a^2 \cos^2 x + b^2 \sin^2 x} =$ [Karnataka CET 2003]

(a) $\frac{\pi}{ab}$ (b) $\frac{\pi}{2ab}$ (c) $\frac{\pi^2}{ab}$ (d) $\frac{\pi^2}{2ab}$

333. The value of the integral $I = \int_0^1 x(1-x)^n dx$ is [AIEEE 2003]

(a) $\frac{1}{n+1}$ (b) $\frac{1}{n+2}$ (c) $\frac{1}{n+1} - \frac{1}{n+2}$ (d) $\frac{1}{n+1} + \frac{1}{n+2}$

334. $\int_0^1 \tan^{-1}\left(\frac{1}{x^2 - x + 1}\right) dx$ is [Orissa JEE 2003]

(a) $\ln 2$ (b) $-\ln 2$ (c) $\frac{\pi}{2} + \ln 2$ (d) $\frac{\pi}{2} - \ln 2$

335. $\int_0^1 \tan^{-1}(1-x+x^2) \, dx =$ [IIT 1998]

(a) $\log 2$ (b) $\log \frac{1}{2}$ (c) $\pi \log 2$ (d) $\frac{\pi}{2} \log \frac{1}{2}$

336. $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx$ is **[AIEEE 2002]**

(a) $\pi^2/4$ (b) π^2 (c) 0 (d) $\pi/2$

337. The value of the integral $\int_{-\pi}^{\pi} \sin mx \sin nx \, dx$ for $m \neq n (m, n \in I)$, is

(a) 0 (b) π (c) $\frac{\pi}{2}$ (d) 2π

338. The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx, a > 0$, is

(a) π (b) $a\pi$ (c) $\frac{\pi}{2}$ (d) 2π

Summation of series by integration

Basic Level

339. $\lim_{n\to\infty} \frac{1^4+2^4+3^4+.....+n^4}{n^5} - \lim_{n\to\infty} \frac{1^3+2^3+3^3+.....+n^3}{n^5}$ **[AIEEE 2003]**

(a) $\frac{1}{30}$ (b) Zero (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

340. $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{\sqrt{n}}{\sqrt{r}(3\sqrt{r}+4\sqrt{n})^2} =$

(a) $\frac{1}{7}$ (b) $\frac{1}{10}$ (c) $\frac{1}{14}$ (d) None of these

341. $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{(r^2 n - m)^{1/3}}{r\, n} =$

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) 0

342. The value of $\lim_{n\to\infty} \left[\frac{(2n)!}{n!\, n^n}\right]^{1/n}$ is equal to

(a) $4e$ (b) $e/4$ (c) $4/e$ (d) None of these

343. $\lim_{n\to\infty} \left[\frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + ... + \frac{1}{n+n}\right]$ is equal to **[Karnataka CET 1993]**

(a) $\log_e 3$ (b) 0 (c) $\log_e 2$ (d) 1

344. $\lim_{n\to\infty} \left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)........\left(1+\frac{n^2}{n^2}\right)\right]^{1/n} =$ .... **[WB JEE 1992, 93]**

(a) $2e^{(\pi+4)/2}$ (b) $2e^{\pi/4-1}$ (c) $2e^{(\pi-4)/2}$ (d) None of these

345. $\lim_{n\to\infty} \frac{1}{n^2} \sum_{r=1}^{n} r e^{r/n} =$ **[EAMCET 1992]**

(a) 0 (b) 1 (c) e (d) $2e$

346. $\lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{2n} \frac{r}{\sqrt{n^2+r^2}}$ equals **[IIT 1997]**

(a) $1+\sqrt{5}$ (b) $-1+\sqrt{5}$ (c) $-1+\sqrt{2}$ (d) $1+\sqrt{2}$

347. $\lim_{n\to\infty}\frac{1}{n}\sum_{r=n+1}^{2n}\log_e\left(1+\frac{r}{n}\right)$ equals

(a) $\log\left(\frac{27}{4e}\right)$ (b) $\log\left(\frac{27}{e^2}\right)$ (c) $\log\left(\frac{4}{e}\right)$ (d) None of these

348. If f is continuous then $\lim_{n\to\infty}\frac{1}{n}\left[f\left(\frac{1}{n}\right)+f\left(\frac{2}{n}\right)+f\left(\frac{3}{n}\right)+....+f\left(\frac{n}{n}\right)\right]$ is nothing but

(a) $\int_0^1 f\left(\frac{1}{x}\right)dx$ (b) $\int_0^1 x\, f(x)dx$ (c) $\int_0^1 \frac{1}{x} f\left(\frac{1}{x}\right)dx$ (d) $\int_0^1 f(x)dx$

349. $\lim_{n\to\infty}\left[\frac{1}{na}+\frac{1}{na+1}+\frac{1}{na+2}+....+\frac{1}{nb}\right]$ is equal to

(a) $\log\left(\frac{b}{a}\right)$ (b) $\log\left(\frac{a}{b}\right)$ (c) $\log a$ (d) $\log b$

350. $\lim_{n\to\infty}\frac{1}{n}\left[\tan\frac{\pi}{4n}+\tan\frac{2\pi}{4n}+....\tan\frac{n\pi}{4n}\right]=$

(a) $\frac{1}{\pi}\log 2$ (b) $\frac{2}{\pi}\log 2$ (c) $\frac{4}{\pi}\log 2$ (d) None of these

351. $\lim_{n\to\infty}\left[\frac{1}{1+n^3}+\frac{4}{8+n^3}+.....\frac{r^2}{r^3+n^3}+......+\frac{1}{2n}\right]=$

(a) $\frac{1}{2}\log 2$ (b) $\frac{1}{3}\log 2$ (c) $\log\frac{1}{2}$ (d) None of these

352. $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\sqrt{\left(\frac{n+r}{n-r}\right)}=$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{2}+1$ (c) π (d) None of these

353. $\lim_{n\to\infty}\left[\frac{1}{\sqrt{(n^2-1)}}+\frac{1}{\sqrt{(n^2-2^2)}}+......\frac{1}{\sqrt{[n^2-(n-1)^2]}}\right]=.......$

(a) 0 (b) $\pi/2$ (c) π (d) None of these

Advance Level

354. If $na=1$ always and $n\to\infty$ then the value of $\Pi\{1+(ar)^2\}^{1/r}$ is

(a) 1 (b) $e^{\pi^2/8}$ (c) $e^{\pi^2/24}$ (d) $e^{-\pi^2/12}$

355. The estimated value of $\frac{1}{1001}+\frac{1}{1002}+\frac{1}{1003}+....+\frac{1}{2000}$ is

(a) 1 (b) $\log_e 3$ (c) $\log_e 2$ (d) None of these

Gamma function

Basic Level

356. $\int_0^{\pi/2}\sin^2 x\cos^3 x\, dx=$ **[Rajasthan PET 1984, 2003]**

(a) 0 (b) $\frac{2}{15}$ (c) $\frac{4}{15}$ (d) None of these

357. The value of $\int_0^{\pi/2}(\sqrt{\sin\theta}\cos\theta)^3 d\theta$ is **[AMU 1999]**

(a) 2/9 (b) 2/15 (c) 8/45 (d) 5/2

358. $\int_{-\pi/2}^{\pi/2}\sin^4 x\cos^6 x\,dx =$ **[EAMCET 2002]**

(a) $\frac{3\pi}{64}$ (b) $\frac{3\pi}{572}$ (c) $\frac{3\pi}{256}$ (d) $\frac{3\pi}{128}$

359. $\int_0^a x^4\sqrt{a^2-x^2}\,dx =$

(a) $\frac{\pi}{32}$ (b) $\frac{\pi}{32}a^6$ (c) $\frac{\pi}{16}a^6$ (d) $\frac{\pi}{8}a^6$

360. The value of $\int_0^{2\pi/3}\cos^4(3x/4)\,dx$ is

(a) $\pi/8$ (b) $9\pi/64$ (c) $9\pi/128$ (d) $\pi/4$

Walli's formula

Basic Level

361. $\int_0^{\pi/2}\sin^5 x\,dx =$

(a) $\frac{8}{15}$ (b) $\frac{4}{15}$ (c) $\frac{8\sqrt{\pi}}{15}$ (d) $\frac{8\pi}{15}$

362. Let $f(x)=\begin{vmatrix}\sec x & \cos x & \sec^2 x+\cot x\,\text{cosec}\, x\\ \cos^2 x & \cos^2 x & \text{cosec}^2 x\\ 1 & \cos^2 x & \cos^2 x\end{vmatrix}$, then $\int_0^{\pi/2} f(x)dx =$ **[IIT 1987]**

(a) $\frac{\pi}{4}+\frac{8}{15}$ (b) $\frac{\pi}{4}-\frac{8}{15}$ (c) $-\frac{\pi}{4}-\frac{8}{15}$ (d) $-\frac{\pi}{4}+\frac{8}{15}$

363. $\int_0^{\pi/2}\sin^{2m} x\,dx =$

(a) $\frac{2m\,!}{(2^m.m\,!)}\cdot\frac{\pi}{2}$ (b) $\frac{(2m)!}{(2^m.m\,!)^2}\cdot\frac{\pi}{2}$ (c) $\frac{2m\,!}{2^m(m\,!)^2}\cdot\frac{\pi}{2}$ (d) None of these

364. $\int_0^{\pi}\cos^3 x\,dx =$ **[Rajasthan 1995; MP PET 1996]**

(a) – 1 (b) 0 (c) 1 (d) π

365. $\int_0^{\pi}\sin^5\left(\frac{x}{2}\right)dx$ equals **[Kurukshetra CEE 1996]**

(a) 16/15 (b) 32/15 (c) 8/15 (d) 5/6

366. The value of $\int_a^{a+(\pi/2)}(\sin^4 x+\cos^4 x)dx$ is

(a) Independent of a (b) $a\left(\frac{\pi}{2}\right)^2$ (c) $\frac{3\pi}{8}$ (d) $\frac{3\pi a^2}{8}$

367. $\int_0^a x(2ax-x^2)^{3/2}dx =$

(a) $a^5\left[\frac{3\pi}{16}-1\right]$ (b) $a^5\left[\frac{3\pi}{16}+1\right]$ (c) $a^5\left[\frac{3\pi}{16}-\frac{1}{5}\right]$ (d) None of these

Leibnitz's rule

Basic Level

368. The least value of the function $F(x)=\int_{5\pi/4}^{x}(3\sin u+4\cos u)du$ on the interval $[5\pi/4, 4\pi/3]$, is

(a) $\sqrt{3}+\frac{3}{2}$ (b) $-2\sqrt{3}+\frac{3}{2}+\frac{1}{\sqrt{2}}$ (c) $\frac{3}{2}+\frac{1}{\sqrt{2}}$ (d) None of these

369. The function $L(x)=\int_1^x \frac{dt}{t}$ satisfies the equation **[IIT 1996; DCE 2001]**

(a) $L(x+y)=L(x)+L(y)$ (b) $L\left(\frac{x}{y}\right)=L(x)+L(y)$ (c) $L(xy)=L(x)+L(y)$ (d) None of these

370. If $\int_{\pi/2}^{x}\sqrt{3-2\sin^2 u}\,du+\int_0^y \cos t\,dt=0$, then $\frac{dy}{dx}=$

(a) $\frac{\sqrt{4-3\sin^2 x}}{\cos y}$ (b) $-\frac{\sqrt{3-2\sin^2 x}}{\cos y}$ (c) $\sqrt{3-2\sin^2 x}+\cos y$ (d) None of these

371. If $\int_0^x f(t)dt = x+\int_x^1 t\,f(t)dt$, then the value of $f(1)$ is **[IIT 1998]**

(a) 1/2 (b) 0 (c) 1 (d) –1/2

372. If $f(t)=\int_{-t}^{t}\frac{dx}{1+x^2}$, then $f'(1)$ is **[Roorkee 2000]**

(a) Zero (b) $\frac{2}{3}$ (c) –1 (d) 1

373. If $f(t)=\int_x^1 \frac{dt}{1+t^2}$ and $I_2=\int_1^{1/x}\frac{dt}{1+t^2}$ for $x > 0$, then

(a) $I_1=I_2$ (b) $I_1>I_2$ (c) $I_2=\cot^{-1}x-\pi/4$ (d) Both (a) and (c)

374. If $\int_0^t \frac{bx\cos 4x-a\sin 4x}{x^2}dx=\frac{a\sin 4t}{t}-1$ where $0<t<\frac{\pi}{4}$, then the value of a, b are equal to

(a) $\frac{1}{4},1$ (b) – 1, 4 (c) 2, 2 (d) 2, 4

375. The equation of tangent to be curve $y=\int_{x^2}^{x^3}\frac{dt}{\sqrt{1+t^2}}$ at $x = 1$ is equal to

(a) $x\sqrt{3}-y+(\sqrt{3}+1)=0$ (b) $x\sqrt{3}-y+1=0$ (c) $x-y\sqrt{2}-1=0$ (d) None of these

376. If $f(x)=\int_2^{x^2}\frac{(\sin^{-1}\sqrt{t})^2}{\sqrt{t}}dt$ then the value of $(1-x^2)\{f''(x)\}^2-2f'(x)$ at $x=\frac{1}{\sqrt{2}}$ is

(a) $2-\pi$ (b) $3+\pi$ (c) $4-\pi$ (d) None of these

377. If $f(x)=\int_0^x t\sin t\,dt$, then $f'(x)=$ **[MNR 1982; Karnataka CET 1999]**

(a) cos *x* + *x* sin *x* (b) *x* sin *x* (c) *x* cos *x* (d) None of these

378. Let $f(x)=\int_1^x \frac{\log_e t}{1+t}dt$, and $f(x)+f\left(\frac{1}{x}\right)=k(\log_e x)^2$, then *k* =

(a) 0 (b) 1 (c) 2 (d) $\frac{1}{2}$

379. The equation $\int_0^x (t^2-8t+13)\,dt = x\sin\left(\frac{a}{x}\right)$ has a solution if $\sin\left(\frac{a}{6}\right)$ is

(a) Zero (b) – 1 (c) 1 (d) None of these

Advance Level

380. The difference between the greatest and least values of the function $\phi(x)=\int_0^x (t+1)\,dt$ on [2, 3] is

(a) 3 (b) 2 (c) 7/2 (d) 11/2

381. If $f(x)=\int_{x^2}^{x^2+1} e^{-t^2}dt$, then *f*(*x*) increases in **[IIT Screening 2003]**

(a) (2, 2) (b) No value of *x* (c) (0, ∞) (d) (– ∞, 0)

382. The value of $\int_0^{n\pi+v} |\sin x|\,dx$ is

(a) $2n+1+\cos v$ (b) $2n+1-\cos v$ (c) $2n+1$ (d) $2n+\cos v$

383. The value of $\int_0^{\sin^4 x} \sin^{-1}\sqrt{t}\,dt+\int_0^{\cos^2 x}\cos^{-1}\sqrt{t}\,dt$ is **[MP PET 2001]**

(a) $\frac{\pi}{2}$ (b) 1 (c) $\frac{\pi}{4}$ (d) None of these

384. The point of extreme of $\int_0^{x^2} \frac{t^2-5t+4}{2+e^t}dt$ are **[IIT Screening]**

(a) $x=-2$ (b) $x=1$ (c) $x=0$ (d) All of these

385. Let $g(x)=\int_0^x f(t)\,dt$ where $\frac{1}{2}\le f(t)\le 1, t\in[0,1]$ and $0\le f(t)\le\frac{1}{2}$ for $t\in(1,2]$, then **[IIT Screening 2000]**

(a) $-\frac{3}{2}\le g(2)<\frac{1}{2}$ (b) $0\le g(2)<2$ (c) $\frac{3}{2}<g(2)\le\frac{5}{2}$ (d) 2 < *g*(2) < 4

386. If $f(x)=\int_0^{\sin x}\cos^{-1}t\,dt+\int_0^{\cos x}\sin^{-1}t\,dt, 0<x<\frac{\pi}{2}$, then $f\left(\frac{\pi}{4}\right)=$

(a) 0 (b) $\pi\sqrt{2}$ (c) 1 (d) $1+\frac{\pi}{2\sqrt{2}}$

387. The function $f(x)=\int_0^x t(t-1)(t-2)dt$ is minimum, when

(a) *x* = 0, 1 (b) *x* = 1, 2 (c) *x* = 0, 2 (d) None of these

Integration with Infinite Function

Basic Level

388. $\int_0^{\infty} \log\left(x+\frac{1}{x}\right)\frac{dx}{1+x^2} =$ **[Rajasthan PET 2000, 2002]**

(a) $\pi \log 2$ (b) $-\pi \log 2$ (c) $\frac{\pi}{2}\log 2$ (d) $\frac{-\pi}{2}\log 2$

389. $\int_0^{\infty} \frac{x\,dx}{(1+x)(1+x^2)} =$ **[Karnataka CET 2003]**

(a) 0 (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) 1

390. Given that $\int_0^{\infty} \frac{x^2dx}{(x^2+a^2)(x^2+b^2)(x^2+c^2)} = \frac{\pi}{2(a+b)(b+c)(c+a)}$ then the value of $\int_0^{\infty} \frac{x^2dx}{(x^2+4)(x^2+9)}$ is **[Karnataka CET 1993]**

(a) $\frac{\pi}{60}$ (b) $\frac{\pi}{20}$ (c) $\frac{\pi}{40}$ (d) $\frac{\pi}{80}$

391. $\int_0^{\infty} \frac{dx}{(x+\sqrt{x^2+1})^3} =$ **[EAMCET 1992]**

(a) $\frac{3}{8}$ (b) $\frac{1}{8}$ (c) $-\frac{3}{8}$ (d) None of these

392. The value of the integral $\int_0^{\infty} \frac{x \log x}{(1+x^2)^2}dx$ is **[Karnataka CET 1997; AMU 2000]**

(a) 0 (b) log 7 (c) 5 log 13 (d) None of these

393. $\int_0^{\infty} \frac{1}{1+e^x}dx =$

(a) log 2 – 1 (b) log 2 (c) log 4 – 1 (d) – log 2

394. $\int_0^{\infty} \frac{\sin(\tan^{-1}x)}{1+x^2}dx$ equals **[Rajasthan PET 1988]**

(a) 0 (b) π (c) 1 (d) $\pi/2$

395. $\int_0^{\infty} \frac{x}{1+x^4}dx$ equals **[Rajasthan PET 1994]**

(a) $\pi/8$ (b) $\pi/4$ (c) $\pi/2$ (d) π

Advance Level

396. The value of the integral $I = \int_1^{\infty} \frac{x^2-2}{x^3\sqrt{x^2-1}}dx$ is

(a) 0 (b) 2/3 (c) 4/3 (d) None of these

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Answer Sheet

Assignment (Basic and Advance Level)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
c	c	c	d	a	c	b	c	c	b	c	c	c	c	b	a	a	d	b	a
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
a	a	c	a	a	a	c	a	b	d	d	d	b	a	a	b	c	b	a	b
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
c	c	a	d	b	a	c	b	a	a	c	b	c	c	b	c	c	c	b	a
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
c	a	a	b	b	b	b	c	a	c	a	c	a	d	b	a	d	a	c	c
81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
d	b	d	b	a	d	b	a	a	c	c	d	b	b	c	b	b	c	b	c
101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
b	d	d	c	c	c	c	b	c	c	d	c	c	b	d	b	a	b	d	d
121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140
b	d	d	b	b	a	b	a	a	d	d	a	b	c	b	a	a	c	d	a
141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160
b	b	c	d	a	b	c	a	b	a	c	d	d	a	b	d	a	b	b	c
161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180
b	d	a	a	a	a	b	d	c	a	c	b	a	c	b	b	c	b	c	c
181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200
c	d	a	b	b	c	d	a	d	d	c	c	b	b	d	a	c	a	d	c
201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220
c	a	d	b	d	d	b	c	a	c	b	b	c	a	d	a	d	d	a	b
221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240
d	b	b	b	b	a	c	d	a	a	b	c	c	b	c	a	d	c	a	d
241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260
c	a	a	d	d	d	a	b	a	c	a	c	c	c	c	a	a	d	d	c
261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280
c	d	b	a	b	a	c	b	b	c	a	c	d	b	c	a	a	a	c	d
281	282	283	284	285	286	287	288	289	290	291	292	293	294	295	296	297	298	299	300
b	d	a	a	d	a	a	c	d	c	b	c	d	c	a	c	b	c	d	c
301	302	303	304	305	306	307	308	309	310	311	312	313	314	315	316	317	318	319	320
a	a	b	a	a	b	c	c	c	c	b	d	c	c	d	b	b	b	a	c
321	322	323	324	325	326	327	328	329	330	331	332	333	334	335	336	337	338	339	340
b	b	d	c	c	d	c	b	b	a	a	d	c	d	a	b	a	c	d	c
341	342	343	344	345	346	347	348	349	350	351	352	353	354	355	356	357	358	359	360
b	c	c	c	b	b	a	d	a	b	b	b	b	c	c	b	c	c	b	d
361	362	363	364	365	366	367	368	369	370	371	372	373	374	375	376	377	378	379	380
a	c	b	b	a	c	c	b	c	b	a	d	d	a	c	d	b	d	c	c
381	382	383	384	385	386	387	388	389	390	391	392	393	394	395	396				

d	b	c	d	b	d	c	a	c	a	a	a	b	c	b	a